

Bidding for a group-specific public-good prize

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Abstract

We examine the equilibrium effort levels of individual players and groups in a contest in which two groups compete with each other to win a group-specific public-good prize, the players choose their effort levels simultaneously and independently, and the winning group is determined by the selection rule of all-pay auctions. We first prove nonexistence of a pure-strategy Nash equilibrium, and then construct a mixed-strategy Nash equilibrium. At the Nash equilibrium, the only active player in each group is a player whose valuation for the prize is the highest in that group; all the other players expend zero effort; and the equilibrium effort levels depend solely on two values – the highest valuation for the prize in each group.

Keywords: Public-good prize; Contest; All-pay auction; Private provision of public goods

JEL classification: D71; D72; D44; H41

1. Introduction

A contest is a situation in which individual players or groups compete with one another by expending irreversible effort to win a prize. In a group contest, if a prize is a public good within a group, we call it a group-specific public-good prize. Contests involving group-specific public-good prizes are easily observed. Consider, for example, a situation in which the government first decides whether to regulate a monopoly and then decides which firm to be the monopolist. In each stage, there is competition. In the first stage, firms – potential monopolists – lobby for the unregulated monopoly while consumer groups lobby for the regulated monopoly.¹ In the second stage, after knowing the government's decision on the form of the monopoly, the firms compete against each other to win the monopoly. The prize for each firm in the first-stage competition is a group-specific public good: if a firm wins its prize, the unregulated monopoly, then all of the firms enjoy being candidates for the unregulated monopoly. The prize for each consumer group is also a group-specific public good: if a consumer group wins its prize, the regulated monopoly, then all consumers enjoy the lower price.

Other examples of contests with group-specific public-good prizes include competition between domestic and foreign firms to obtain governmental trade policies favorable to them, R&D competition between consortiums, election campaigns between political parties, and competition between local governments to invite business firms into their districts.

The purpose of this paper is to examine the equilibrium effort levels of individual players and groups in such contests. To do so, we consider the following contest. Two groups compete with each other to win a group-specific public-good prize. The individual players choose their effort levels simultaneously and independently. Each player's effort is irreversible. The winning group is determined by the selection rule of all-pay auctions – a group which expends more effort (or submits a higher group-bid) than its rival wins the

prize with certainty. Each player's valuation for the prize is publicly known and the valuations may differ across the individual players.

We first show that no pure-strategy Nash equilibrium exists. Then, constructing a mixed-strategy Nash equilibrium, we show that the equilibrium effort levels of individual players and groups depend solely on two values – the highest valuation for the prize in each group. This implies that the equilibrium effort levels are independent of the number of players, the sum of valuations, and the distribution of valuations in each group, as long as changes in these do not change the highest valuation for that group. We also show that, at the mixed-strategy Nash equilibrium, there are only two active players, one for each group; each active player is one who has the highest valuation for the prize in his group; and all the other players except these two expend zero effort.² We argue that the free-rider problem occurs at the equilibrium since, in each group, a highest-valuation player obtains the greatest gross marginal payoff, while all members including him experience the same marginal cost.

Katz et al. (1990), Ursprung (1990), Baik (1993), Riaz et al. (1995), and Baik and Shogren (1998) also study contests with group-specific public-good prizes. Among them, Baik (1993) and Baik and Shogren (1998) are closely related to this paper. The main difference is that their rules of selecting the winning group differ from the rule in this paper – their probability-of-winning functions are continuous while our function is discontinuous. According to our selection rule, the group which expends the largest effort wins the prize with certainty. Interestingly, however, the main result in Baik (1993) and Baik and Shogren (1998) is similar to ours: the equilibrium effort levels of individual players and groups depend solely on two values – the highest valuation for the prize in each group.

Hillman and Riley (1989) and Baye et al. (1996) consider a contest in which many *individual* players compete with one another by expending irreversible effort to win a *private-good* prize, and the winner is determined by the selection rule of all-pay

auctions – a player who submits the highest *individual* bid wins the prize with certainty. They show that no pure-strategy Nash equilibrium exists.³ They also show that, if top two players (according to valuation for the prize) have *greater* valuations than the third, then there exists a unique mixed-strategy Nash equilibrium at which only the top two players are active and all the other players expend zero effort (or bid zero) with probability one. Note that we also obtain the two-active-player result. In this paper, however, the two active players are not top two players (according to valuation) in the contest. They consist of a highest-valuation player *in each group*.

This paper is related to the literature on the private (also called, voluntary) provision of public goods – the literature which deals with situations in which public goods are financed by voluntary contributions of individuals (see, for example, Olson, 1965; Olson and Zeckhauser, 1966; Bergstrom et al., 1986; Gradstein et al., 1994; Varian, 1994; Vicary, 1997; Boadway and Hayashi, 1999). Indeed, in this paper, the players in each group play a game of the private provision of a public good – they choose their effort levels noncooperatively to win their public-good prize. We show that, as is often the case with the private-public-goods-provision literature, the free-rider problem arises. The free-rider problem in this paper is the severest form in that, in each group, all the players except a highest-valuation player are free riders.

This paper is also related to the literature on mechanism design, particularly, designing mechanisms for the provision of public goods. Papers in this literature include Groves and Ledyard (1977), Jackson and Moulin (1992), Kleindorfer and Sertel (1994), Bag and Winter (1999), Deb and Razzolini (1999), and Saijo and Yamato (1999). Based on our model, one could propose the "all-pay-auction mechanism" for the provision of group-specific public goods. According to the mechanism, the government auctions off a group-specific public good;⁴ all individual players in participating groups or communities submit their bids noncooperatively; the government collects all the bids and provides the public good for the group which submits the highest group-bid for the public good; and the

collected money goes to the general fund of government revenues. The main idea of this proposed mechanism is to finance a group-specific public good partly or fully by auctioning it off.

The paper proceeds as follows. Section 2 develops the model. Section 3 first proves the nonexistence of a pure-strategy Nash equilibrium, and then finds a mixed-strategy Nash equilibrium at which there is just one active player in each group. Section 4 presents two modified models in which there are many active players in equilibrium. Finally, Section 5 offers our conclusions.

2. The model

Consider a contest in which two groups, 1 and 2, compete with each other for a prize. Group i consists of m_i risk-neutral players who expend effort to win the prize, where $m_i \geq 1$. Each player's effort is irreversible – each player cannot recover his effort expended whether or not his group wins the prize. Let x_{ik} represent the effort level expended by player k in group i and let X_i represent the effort level expended by all the players in group i , so that $X_i = \sum_{k=1}^{m_i} x_{ik}$. Effort levels are nonnegative and are measured in units commensurate with the prize. The winning group is determined by the selection rule of all-pay auctions. Let $p_i(X_1, X_2)$ denote the probability that group i wins the prize when the groups' effort levels are X_1 and X_2 . The probability-of-winning function (also called, the contest success function) for group i is then:

$$\begin{aligned}
 p_i(X_1, X_2) = & \quad 1 & \quad \text{if } X_i > X_j \\
 & \quad 1/2 & \quad \text{if } X_i = X_j \\
 & \quad 0 & \quad \text{if } X_i < X_j,
 \end{aligned} \tag{1}$$

where $i \neq j$.⁵ (Throughout the paper, when we use i and j at the same time, we mean that $i \neq j$.) This probability-of-winning function implies that, if a group expends more effort than its rival, it wins the prize with certainty. Given X_j , this function is discontinuous at $X_i = X_j$.

The prize is a public good for the players in each group. Valuations for the group-specific public-good prize may differ across the individual players. Each player's valuation for the prize is positive and publicly known. Let v_{ik} represent the valuation for the prize of player k in group i .

Assumption 1. *Without loss of generality, we assume that $v_{is-1} \geq v_{is} > 0$ for $s = 2, \dots, m_i$.*

Let π_{ik} represent the expected payoff for player k in group i . We have then

$$\pi_{ik} = v_{ik}P_i(X_1, X_2) - x_{ik}. \quad (2)$$

Although the players in each group have the same goal of winning the group-specific public-good prize, they choose their effort levels *independently*. Assume that all the players in the contest choose their effort levels simultaneously – that is, when a player chooses his effort level, he does not know the other players' effort levels. We assume that all of the above is common knowledge among the players. We employ Nash equilibrium as the solution concept.

3. A mixed-strategy Nash equilibrium involving free riders

This section first proves the nonexistence of a pure-strategy Nash equilibrium, and then constructs a Nash equilibrium in mixed strategies. We begin by deriving the best response of player k in group i – the effort level which maximizes his expected

payoff – given the other players' effort levels. Let $X_i(-k)$ denote the total effort expended by group i 's players except player k : $X_i(-k) \equiv X_i - x_{ik}$.

Lemma 1. *Given the other players' effort levels, the best response of player k in group i is*

$$x_{ik}^B = \begin{cases} X_j - X_i(-k) + \epsilon & \text{if } 0 \leq X_j - X_i(-k) < v_{ik} \\ 0 & \text{otherwise,} \end{cases}$$

where x_{ik}^B is his best response and ϵ is an infinitesimally small positive.

Proof. First, consider the case where $0 \leq X_j - X_i(-k) < v_{ik}$. If player k in group i expends effort which is less than $X_j - X_i(-k)$, his group's effort level is less than the other group's. Then, according to (1), his group loses the prize. It follows from (2) that his expected payoff is negative. If player k in group i expends effort of $X_j - X_i(-k)$, his group's effort level is equal to the other group's. The probability that his group wins the prize is one half and thus his expected payoff may be positive but is not maximized. If player k in group i expends effort of $X_j - X_i(-k) + \epsilon$, his group wins the prize with certainty and thus his expected payoff is maximized.

Next, consider the case where $X_j - X_i(-k) \geq v_{ik}$. For any positive effort level from player k in group i , his expected payoff is negative. However, if he expends zero effort, his expected payoff is zero. Therefore, expending zero effort is his best response.

Finally, in the case where $X_j - X_i(-k) < 0$, his best response is zero since his group wins the prize without any effort from him. \square

Using Lemma 1, we obtain the following proposition.

Proposition 1. *There exists no Nash equilibrium in pure strategies.*

Proof. Suppose on the contrary that there is a pure-strategy Nash equilibrium, denoted by the $(m_1 + m_2)$ -tuple vector of effort levels, $(x_{11}^N, \dots, x_{1m_1}^N, x_{21}^N, \dots, x_{2m_2}^N)$. Then, since each player's equilibrium effort level is the best response to the other players' equilibrium effort levels, using Lemma 1, we have:

$$x_{ik}^N = X_j^N - X_i^N(-k) + \epsilon \quad \text{if } 0 \leq X_j^N - X_i^N(-k) < v_{ik} \quad (3)$$

$$0 \quad \text{otherwise.} \quad (4)$$

First, consider the case where $X_i^N = 0$. Since every player in group i expends zero effort, it follows from (4) that $X_j^N \geq v_{i1}$. On the other hand, (3) says that, if a group's equilibrium effort level is zero, then the other group's equilibrium effort level must be ϵ . This implies that in the present case $X_j^N = \epsilon$ must hold. Hence, we have both $X_j^N \geq v_{i1}$ and $X_j^N = \epsilon$. This leads to a contradiction, since ϵ is an infinitesimally small positive and thus is smaller than v_{i1} .

Next, consider the case where $X_1^N > 0$ and $X_2^N > 0$. We have: $x_{ih}^N = X_j^N - X_i^N(-h) + \epsilon$ for some h in group i (see (3)). Using this, we obtain both $X_1^N = X_2^N + \epsilon$ and $X_2^N = X_1^N + \epsilon$. This leads to a contradiction.

Therefore, there exists no Nash equilibrium in pure strategies. \square

Next, we construct a Nash equilibrium in mixed strategies. At the mixed-strategy Nash equilibrium, as we will see, the only "active" player in each group is a player whose valuation for the prize is the highest in that group. All the other players except these two expend zero effort and free ride. The equilibrium mixed strategy for each active player – and thus his equilibrium *expected* effort level – is the same as that resulting in the contest in which only those two active players compete to win the prize.

Let $G_{ik}(x_{ik})$ denote a cumulative distribution function of x_{ik} – a mixed strategy of player k in group i – and let $g_{ik}(x_{ik})$ denote the corresponding probability density function of x_{ik} . The following lemma is useful in obtaining our main result, Proposition 2.

Lemma 2. *Suppose that player h in group i and player d in group j with $v_{ih} \geq v_{jd}$ play the mixed strategies, $G_{ih}(x_{ih}) = x_{ih}/v_{jd}$ for $x_{ih} \in [0, v_{jd}]$ and $G_{jd}(x_{jd}) = (v_{ih} - v_{jd} + x_{jd})/v_{ih}$ on the support, $[0, v_{jd}]$, respectively. Then, when the other players (except these two active players) use the pure strategy of 0, (a) the best response of player t (for $t \neq h$) in group i is positive if $v_{it} > v_{ih}$ and zero if $v_{it} \leq v_{ih}$, and (b) the best response of player z (for $z \neq d$) in group j is positive if $v_{jz} > v_{jd}$ and zero if $v_{jz} \leq v_{jd}$.*

Proof. First, we prove part (a). The best response of player t in group i is a value of x_{it} which maximizes

$$\pi_{it} = v_{it}\Pr[X_{ih} + x_{it} > X_{jd}] - x_{it},$$

where X_{ih} and X_{jd} represent random variables with the cumulative distribution functions, $G_{ih}(\cdot)$ and $G_{jd}(\cdot)$, respectively. Since $\Pr[X_{ih} + x_{it} > X_{jd}] = 1 - (v_{jd} - x_{it})^2/2v_{ih}v_{jd}$ (see Appendix A), we have $\pi_{it} = v_{it}[1 - (v_{jd} - x_{it})^2/2v_{ih}v_{jd}] - x_{it}$. The first-order condition for maximizing π_{it} is $d\pi_{it}/dx_{it} = v_{it}(v_{jd} - x_{it})/v_{ih}v_{jd} - 1 = 0$. Since π_{it} is strictly concave in x_{it} and $x_{it} \geq 0$, the best response of player t in group i is then $v_{jd}(1 - v_{ih}/v_{it})$ if $v_{it} > v_{ih}$ and zero if $v_{it} \leq v_{ih}$.

Next, we prove part (b). The best response of player z in group j is a value of x_{jz} which maximizes

$$\pi_{jz} = v_{jz}\Pr[X_{jd} + x_{jz} > X_{ih}] - x_{jz}$$

subject to

$$0 \leq x_{jz} \leq v_{jd}.^6$$

Since $\Pr[X_{jd} + x_{jz} > X_{ih}] = [v_{jd}^2 + (2v_{ih} - x_{jz})x_{jz}]/2v_{ih}v_{jd}$ (see Appendix B), we have $\pi_{jz} = v_{jz}\{[v_{jd}^2 + (2v_{ih} - x_{jz})x_{jz}]/2v_{ih}v_{jd}\} - x_{jz}$. The first-order condition for maximizing π_{jz} (without the constraint) is $d\pi_{jz}/dx_{jz} = v_{jz}(v_{ih} - x_{jz})/v_{ih}v_{jd} - 1 = 0$. Since π_{jz} is strictly concave in x_{jz} and $0 \leq x_{jz} \leq v_{jd}$, the best response of player z in group j is then $\min\{v_{ih}(1 - v_{jd}/v_{jz}), v_{jd}\}$ if $v_{jz} > v_{jd}$ and zero if $v_{jz} \leq v_{jd}$. \square

Using Lemma 2, we obtain the following proposition.

Proposition 2. *Without loss of generality, assume that $v_{11} \geq v_{21}$. Then the following strategy profile is a mixed-strategy Nash equilibrium of the game. (a) Player 1 in group 1 plays the mixed strategy, $G_{11}^*(x_{11}) = x_{11}/v_{21}$ for $x_{11} \in [0, v_{21}]$. (b) Player 1 in group 2 plays the mixed strategy, $G_{21}^*(x_{21}) = (v_{11} - v_{21} + x_{21})/v_{11}$ on the support, $[0, v_{21}]$. (c) All the other players use the pure strategy of 0: $x_{1s}^* = 0$ for $s = 2, \dots, m_1$ and $x_{2s}^* = 0$ for $s = 2, \dots, m_2$.*

Proof. The proof concerning parts (a) and (b) is well known. Since only two players are active, their equilibrium strategies must be the same as those resulting in the first-price all-pay auction with two bidders. See Hillman and Riley (1989) or Hirshleifer and Riley (1992, pp. 377-379) for the derivation of the Nash equilibrium of the first-price all-pay auction with two bidders. The proof concerning part (c) is done by Lemma 2. Since, by Assumption 1, $v_{i1} \geq v_{is}$ for $s = 2, \dots, m_i$, Lemma 2 implies that, given the other players' strategies specified above, the best response of player s in group i is zero. \square

At the mixed-strategy Nash equilibrium, there are only two active players, one for each group. Each active player has the highest valuation for the prize in his own group. (Recall that player 1 in each group is one of the highest-valuation players in that group.)

All the other players except these two expend zero effort and free ride. Note that, since player 1 in group 2 expends positive effort with probability v_{21}/v_{11} , it is possible that only player 1 in group 1 – a player with the highest valuation in the contest – *actually* expends positive effort. Note also that, even though each active player has the highest valuation for the prize in his own group, his equilibrium expected payoff can be less than other members' due to the free-rider problem. In other words, the "exploitation" of the highest-valuation player by other members may occur.⁷

Why does the free-rider problem occur at the equilibrium? A plausible explanation is that, in each group, a highest-valuation player obtains the greatest gross marginal payoff, while all members including him experience the same marginal cost. Given that the two groups' effort levels are equal, when a group's effort level increases by ϵ , the players with the highest valuation benefit most. Thus one of the hungriest players exerts effort while the others wait for him to do so.

The equilibrium mixed strategy for each active player is the same as that resulting in the contest in which only those two active players compete to win the prize. This implies that the number of players, the sum of valuations, and the distribution of valuations in each group only affect the equilibrium effort levels of individual players and groups if changes in them lead to a change in the highest valuation for that group. This also implies that, to obtain the equilibrium effort levels, we only need to solve a reduced game in which only two players, a highest-valuation player in group 1 and that in group 2, compete to win the prize.

Using probability-of-winning functions different from ours, Baik (1993) and Baik and Shogren (1998) obtain similar results. Baik (1993) considers a contest in which n groups compete with one another to win a group-specific public-good prize, the individual players choose their effort levels simultaneously and independently, and the probability-of-winning function for group i takes a continuous-function form: $p_i = p_i(X_1, \dots, X_n)$, where X_i represents the effort level expended by all the players in group i . In Baik and Shogren

(1998), two groups compete and the winning group is determined by a general difference-form probability-of-winning function.

Proposition 3 describes another interesting result.

Proposition 3. *No mixed-strategy Nash equilibrium exists at which there are only two active players, one for each group, but only one or neither of them is a highest-valuation player in his group.*

The proof of Proposition 3 is immediate from Lemma 2. If an active player is not a highest-valuation player in his group, then a highest-valuation player in that group has an incentive to deviate – he has an incentive to increase his effort level.

4. Modified models and many active players in equilibrium

We have shown in Section 3 that, in equilibrium, there is just one active player in each group. In this section, we present two modified models in which there are many active players in equilibrium. The first contains a sequential version of the original game and the second contains a game with collective group-bid decisions.

4.1. The sequential version of the original game

Consider a game in which the players in group 1 first choose their effort levels and then, after observing them, the players in group 2 choose their effort levels. The players in each group choose their effort levels simultaneously and independently. Thus this game contains both sequential and simultaneous parts.

We are interested in the players' effort levels specified in a subgame-perfect equilibrium of the game. Let us denote them by the $(m_1 + m_2)$ -tuple vector of effort levels, $(x_{11}^S, \dots, x_{1m_1}^S, x_{21}^S, \dots, x_{2m_2}^S)$. Let V_i represent the sum of the valuations of the

players in group i , so that $V_i = \sum_{k=1}^{m_i} v_{ik}$. Suppose that $V_1 \geq V_2$. (In Section 4, we do not assume Assumption 1.) Then it is straightforward to obtain: (a) $0 \leq x_{1k}^S \leq v_{1k}$ for $k = 1, \dots, m_1$, (b) $x_{11}^S + \dots + x_{1m_1}^S = X_1^S = V_2$, and (c) $x_{2k}^S = 0$ for $k = 1, \dots, m_2$.⁸

It follows from (a) and (b) that, in such an equilibrium, player k in group 1 expends

$$x_{1k}^S = \begin{cases} V_2 - X_1^S(-k) & \text{if } 0 \leq V_2 - X_1^S(-k) \leq v_{1k} \\ 0 & \text{otherwise,} \end{cases}$$

where $X_1^S(-k) \equiv X_1^S - x_{1k}^S$. Therefore, it is easy to see that the game has numerous equilibria in which many players in group 1 are active – equilibria in which they expend positive effort. Furthermore, it has equilibria in which all the players in group 1 are active.

4.2. The game with collective group-bid decisions

Consider a game in which the players in each group jointly choose their group-bid, and then they each choose their own contributions to the group-bid. Formally, we consider the following four-stage game. In the first stage, trying to maximize the sum total of their expected payoffs, the players in each group jointly choose their group-bid. In the second stage, knowing their group-bid but *not* the other group's, the players in each group choose their own contributions to their group-bid simultaneously and independently. In the third stage, if the sum of their individual contributions is greater than or equal to their predetermined group-bid, the players in each group submit the group-bid; otherwise, they bid zero. In any case, players' contributions are not refunded. The money left over, if any, is used for other purposes. In the final stage, the winning group is determined according to (1). We assume that all of the above is common knowledge among the players. We employ subgame-perfect equilibrium as the solution concept.

Let Π_i represent the sum of the expected payoffs for the players in group i . Using (2), we obtain:

$$\Pi_i = \sum_{k=1}^{m_i} \pi_{ik} = V_i p_i(X_1, X_2) - X_i.$$

In the first stage, since the players in group i collectively choose their group-bid, X_i , and try to maximize the sum of their expected payoffs, Π_i , we treat them as one strategic player – simply called group i – whose expected payoff is given by Π_i . Then the first-stage competition becomes the first-price all-pay auction in which group 1 and group 2 participate. Let $H_i(X_i)$ denote a cumulative distribution function of X_i – a mixed "strategy" of group i . Without loss of generality, assume that $V_1 \geq V_2$. Let B_i represent the group i 's group-bid *chosen* in the first stage. Then the full game has the following equilibrium: (a) group 1 plays the mixed strategy, $H_1^*(X_1) = X_1/V_2$ for $X_1 \in [0, V_2]$ and group 2 plays the mixed strategy, $H_2^*(X_2) = (V_1 - V_2 + X_2)/V_1$ on the support, $[0, V_2]$, and (b) player k in group i contributes $x_{ik}^C = (v_{ik}/V_i)B_i$, where $B_i \in [0, V_2]$.

The proof concerning part (a) is well known. As mentioned above, the first-stage intergroup competition is just the standard first-price all-pay auction with two bidders whose valuations are V_1 and V_2 . Then, one can refer to Hillman and Riley (1989) or Hirshleifer and Riley (1992, pp. 377-379) for the derivation of the Nash equilibrium of the first-price all-pay auction with two bidders.

Next, for the proof concerning part (b), we will show that, in their intragroup contribution "game," the players in each group have no incentive to deviate from their contributions specified in part (b). Consider first player k in group 1. Given part (a), his expected payoff is given by $\theta_{1k} = v_{1k}H_2^*(B_1) - x_{1k} = v_{1k}(V_1 - V_2 + B_1)/V_1 - x_{1k}$, when he contributes x_{1k} and group 1 submits the predetermined group-bid B_1 , and $\theta_{1k} = -x_{1k}$, when he contributes x_{1k} and group 1 bids zero. Then, given the contributions of the other players in group 1, $(x_{11}^C, \dots, x_{1k-1}^C, x_{1k+1}^C, \dots, x_{1m_1}^C)$, his expected payoff is equal to

$v_{1k}(V_1 - V_2)/V_1$ if he contributes x_{1k}^C , it is less than $v_{1k}(V_1 - V_2)/V_1$ if he contributes more than x_{1k}^C , and it is less than or equal to zero if he contributes less than x_{1k}^C , and thus x_{1k}^C is his best response. It implies that, given part (a), the players in group 1 have no incentive to deviate from their contributions specified in part (b). Next, consider player k in group 2. Given part (a), his expected payoff is $\theta_{2k} = v_{2k}H_1^*(B_2) - x_{2k} = v_{2k}B_2/V_2 - x_{2k}$, when he contributes x_{2k} and group 2 submits the predetermined group-bid B_2 , and $\theta_{2k} = -x_{2k}$, when he contributes x_{2k} and group 2 bids zero. Then, given the contributions of the other players in group 2, $(x_{21}^C, \dots, x_{2k-1}^C, x_{2k+1}^C, \dots, x_{2m_2}^C)$, his expected payoff is equal to zero if he contributes x_{2k}^C , it is less than zero if he contributes more than x_{2k}^C , and it is less than or equal to zero if he contributes less than x_{2k}^C . It means that, given part (a), the players in group 2 have no incentive to deviate from their contributions specified in part (b).

In the above equilibrium of the game, all the players in the contest are active. The equilibrium contribution of player k in group i is equal to $(v_{ik}/V_i)B_i$ when his group actually submits a bid of B_i .

5. Conclusions

We have examined the equilibrium effort levels of individual players and groups in a contest in which two groups compete with each other to win a group-specific public-good prize and the individual players choose their effort levels simultaneously and independently. We have modeled the contest as a first-price all-pay auction: each individual player's effort is irreversible, a group which expends more effort than its rival wins the prize with certainty, and the winning group pays the higher "bid," i.e., its own bid.

We have first shown that no pure-strategy Nash equilibrium exists, and then we have constructed a Nash equilibrium in mixed strategies. At the mixed-strategy Nash equilibrium, (a) if a player expends positive effort, he has the highest valuation for the prize in his group; (b) a player whose valuation for the prize is less than the valuation of another player in his group expends zero effort and free rides; and (c) the equilibrium effort

levels of individual players and groups depend solely on the highest valuation for the prize in group 1 and that in group 2. Therefore, in this case, the equilibrium effort levels are independent of the number of players, the sum of valuations, and the distribution of valuations in each group, as long as changes in these do not change the highest valuation for that group. This implies that a new member only changes the equilibrium effort levels if he is hungrier than all the existing members.

Are there other types of mixed-strategy Nash equilibria in our (main) model? Can one obtain a mixed-strategy Nash equilibrium similar to ours when there are more than two groups? These are interesting questions. We leave them for future research.

Footnotes

1. Let us suppose, as in Baik (1999), the monopoly profits are greater with the unregulated monopoly than with the regulated monopoly, while consumer surplus is greater with the regulated monopoly than with the unregulated monopoly.
2. Based on the results, one may say that our contest with a group-specific public-good prize is a "best-shot" contest – a contest in which collective effort of each group is just the effort of a highest-valuation player in the group. We borrow the term *best-shot* from Hirshleifer (1983, 1985) who discusses three ways of determining the socially available amount of a public good: the summation rule, the weakest-link rule, and the best-shot rule.
3. We also show that no pure-strategy Nash equilibrium exists. However, the proof of the result is quite different from that in Hillman and Riley (1989) and Baye et al. (1996). The reason is that, in this paper, the winner is determined by the selection rule using *group-bids*, not individual bids, and the prize is a group-specific *public good*, not a private good.
4. Consider, for example, the case where the government uses an all-pay auction to designate the location of a government institution, a government-owned corporation, or a new highway.
5. This probability-of-winning function is extensively used in the literature on the theory of contests. Examples include Hillman and Samet (1987), Hillman and Riley (1989), Ellingsen (1991), Hirshleifer and Riley (1992), Baye et al. (1993, 1996), Amann and Leininger (1996), Krishna and Morgan (1997), Che and Gale (1997, 1998), and Clark and Riis (1998). For other forms of probability-of-winning functions, see Baik (1998).
6. We introduce the constraint, $0 \leq x_{jz} \leq v_{jd}$, because $\Pr[X_{jd} + x_{jz} > X_{ih}] = 1$ for $x_{jz} \geq v_{jd}$ and thus a value of x_{jz} greater than v_{jd} is not the player's best response.

7. Olson (1965, pp. 3, 29, 35) coined the phrase *the exploitation of the great by the small*. In the context of Olson and Zeckhauser (1966), it means that a larger country contributes more to international public goods than a smaller country and furthermore a larger country contributes more than proportionately. Or, more strictly, it means that, due to their disproportionate contributions to international public goods, the net payoffs to a larger country are less than those to a smaller country. We thank one of the referees for introducing the phrase to us.

8. Other types of subgame-perfect equilibria may exist. For example, a subgame-perfect equilibrium may exist in which we have $x_{1k}^S = 0$ for $k = 1, \dots, m_1$, and $X_2^S = x_{2h}^S = \epsilon$ for some h in group 2. We ignore them for concise exposition.

Appendix A

We show that $\Pr[X_{ih} + x_{it} > X_{jd}] = 1 - (v_{jd} - x_{it})^2/2v_{ih}v_{jd}$. Note that X_{ih} and X_{jd} are independent random variables since all the players in the contest choose their effort levels independently.

$$\begin{aligned}
& \Pr[X_{ih} + x_{it} > X_{jd}] \\
&= \int_{-\infty}^{\infty} g_{ih}(x_{ih})G_{jd}(x_{ih} + x_{it})dx_{ih} \\
&= \int_{-\infty}^{\infty} (1/v_{jd})I_{[0, v_{jd}]}(x_{ih}) \\
&\quad \times \{[(v_{ih} - v_{jd} + x_{ih} + x_{it})/v_{ih}]I_{[0, v_{jd}-x_{it}]}(x_{ih}) + I_{(v_{jd}-x_{it}, v_{jd}]}(x_{ih})\}dx_{ih} \\
&= \int_0^{v_{jd}-x_{it}} (1/v_{jd})[(v_{ih} - v_{jd} + x_{ih} + x_{it})/v_{ih}]dx_{ih} + \int_{v_{jd}-x_{it}}^{v_{jd}} (1/v_{jd})dx_{ih} \\
&= (1/v_{ih}v_{jd})[(v_{ih} - v_{jd} + x_{it})x_{ih} + x_{it}^2/2]_0^{v_{jd}-x_{it}} + [x_{ih}/v_{jd}]_{v_{jd}-x_{it}}^{v_{jd}} \\
&= (1/v_{ih}v_{jd})[(v_{ih} - v_{jd} + x_{it})(v_{jd} - x_{it}) + (v_{jd} - x_{it})^2/2] + 1 - (v_{jd} - x_{it})/v_{jd} \\
&= 1 - (v_{jd} - x_{it})^2/2v_{ih}v_{jd},
\end{aligned}$$

where $I_A(\cdot)$ is the indicator function of a set A .

Appendix B

We show that $\Pr[X_{jd} + x_{jz} > X_{ih}] = [v_{jd}^2 + (2v_{ih} - x_{jz})x_{jz}]/2v_{ih}v_{jd}$. Note that X_{jd} and X_{ih} are independent random variables since all the players in the contest choose their effort levels independently.

$$\Pr[X_{jd} + x_{jz} > X_{ih}]$$

$$\begin{aligned} &= \int_{-\infty}^{\infty} g_{jd}(x_{jd})G_{ih}(x_{jd} + x_{jz})dx_{jd} \\ &= \int_{-\infty}^{\infty} [(1 - v_{jd}/v_{ih})I_{\{0\}}(x_{jd}) + (1/v_{ih})I_{(0, v_{jd}]}(x_{jd})] \\ &\quad \times \{[(x_{jd} + x_{jz})/v_{jd}]I_{[0, v_{jd}-x_{jz}]}(x_{jd}) + I_{(v_{jd}-x_{jz}, v_{jd}]}(x_{jd})\}dx_{jd} \\ &= (1 - v_{jd}/v_{ih})(x_{jz}/v_{jd}) + \int_0^{v_{jd}-x_{jz}} (1/v_{ih})[(x_{jd} + x_{jz})/v_{jd}]dx_{jd} + \int_{v_{jd}-x_{jz}}^{v_{jd}} (1/v_{ih})dx_{jd} \\ &= (1 - v_{jd}/v_{ih})(x_{jz}/v_{jd}) + (1/v_{ih}v_{jd})\int_0^{v_{jd}-x_{jz}} [(x_{jd} + x_{jz})]dx_{jd} + \int_{v_{jd}-x_{jz}}^{v_{jd}} (1/v_{ih})dx_{jd} \\ &= (1 - v_{jd}/v_{ih})(x_{jz}/v_{jd}) + (1/v_{ih}v_{jd})\left[x_{jd}^2/2 + x_{jz}x_{jd}\right]_0^{v_{jd}-x_{jz}} + \left[x_{jd}/v_{ih}\right]_{v_{jd}-x_{jz}}^{v_{jd}} \\ &= [v_{jd}^2 + (2v_{ih} - x_{jz})x_{jz}]/2v_{ih}v_{jd}, \end{aligned}$$

where $I_A(\cdot)$ is the indicator function of a set A .

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