INTRODUCTION

"Μή μου τοὺς κύκλους τάραττε."

The words "do not disturb my circles" are said to be Archimedes' last before he was slain by a Roman soldier in the tumult of the pillaging of Syracuse. The timeless tranquil eternity of the not-to-be-disturbed circles in the midst of this account of hurly-burly and death is emblematic of the contrast between mathematics and stories: history, legends, anecdotes, and narratives of all sorts thrive on drama, on motion and confusion, while mathematics requires a clarity of thought that, in many instances, comes only after prolonged quiet reflection. At first glance, then, it might seem that mathematics and narrative have little use for each other, but this is not so. As anyone who teaches the subject knows well, the appropriate narrative helps make its substance more comprehensible, while the lack of a narrative frame may render mathematics indigestible or even, at times, downright incomprehensible.

This dependence of mathematics on narrative is not surprising: after all, mathematics is created by people, and people live, grow, think, and create stories. Stories play crucial roles in our discovering, creating, explaining, and organizing knowledge, and thus mathematics also has a great need for narrative, even though its taste for general ideas might make one forget this. Yet despite this interdependence of mathematics and narrative, until the last few decades there was little attempt to examine the connections between the two domains. Apart from the more traditional source of mathematics-related narratives, the historical and biographical narratives of the development of the field, these connections are revealed in accounts focusing on the drama of the motivations and aspirations of the creators of mathematics, whether such accounts are expressed as dreams, quests, or stories of other kinds. Mathematics does not live in splendid abstraction and isolation. A close reading of certain mathematical treatises with a view to their characteristics as narratives reveals the troubled self-questioning of their authors, the drama, and the false moves that accompany the actual process of research. A full understanding of the enterprise of mathematics requires an awareness of the narrative aspects intrinsic to it. Going the other way, scholars studying narratalogical forms often are helped by adopting a mathematical way of thinking to discover the forms' underlying intricate structure. A simple example is the complexity of referents to time in this passage from Marcel Proust's *Jean Santeuil*, that prompted a mathematical analysis by Genette[1]:

> Sometimes passing in front of the hotel he remembered the rainy days when he used to bring his nursemaid that far, on a pilgrimage. But he remembered them without the melancholy that he then thought he would surely some day savor on feeling that he no longer loved her. For this melancholy, projected in anticipation prior to the indifference that lay ahead, came from his love. And this love existed no more.

Happily, in the last few decades much intellectual activity has been trained on the overlap of mathematics and narrative, as manifested in the proliferation of works of fiction and narrative nonfiction that take their subject matter from the world of mathematical research. Mathematicians watch with delight, often tinged with disbelief, as endeavors that were until recently largely unknown, and totally arcane to non-mathematicians, such as research on Fermat's last theorem, the Riemann hypothesis or the Poincaré conjecture, become the subject of best-selling books or feature in novels, plays, and films whose plots are set in the world of mathematics, both real and fictional.

More important for this book, the interplay of mathematics and narrative is also becoming the subject of theoretical exploration. Historians, philosophers, cognitive scientists, sociologists, and literary theorists, as well as scholars in other branches of the humanities, are now venturing into this previously dark territory, making new discoveries and contributions. Such theoretical exploration of the old yet new connection between mathematics and narrative is the unifying theme of the fifteen essays in this volume, written by scholars from various disciplines. Many of the essays are original contributions in more than one sense of the word as their authors open up new directions of research.

More precisely, six of the essays deal in various ways with the history of mathematics, a discipline that, though it has mathematical ideas at its center, is narrative in form. None of these contributions follows the older approaches, either internalist, in which progress in a scientific field is interpreted solely within its own bounds, or Whig-historical, which sees in the past of mathematics merely a precursor to the mathematics of today. Without disregarding the underlying story of the development of mathematical ideas and techniques, the contributors look at the history of mathematics in more complex ways. Whether they are investigating mathematical people or mathematical ideas or, more often, the two intertwined, their work aims at relating the creation of mathematics within a larger frame, whether historical or biographical, personal or cultural.

A good example of this approach is Amir Alexander's "From Voyagers to Martyrs: Toward a Storied History of Mathematics." "Storied history" may seem redundant, but in Alexander's treatment it certainly is not, especially when "storied" is read as antithetical to "internalist." Writing from a modern tendency in the historiography of mathematics, Alexander presents the history of mathematics almost as tableaux of stories that are structured according to underlying narrative patterns. In his account, these stories are not merely the retelling of events that once happened but play a crucial role in forming the meaning of such events. Alexander's thesis is that the transformation that occurred in mathematics in the late eighteenth and nineteenth centuries was at least partly guided by the form of the underlying stories told about it. Of particular note is the transition from the older narratives, originating in the Renaissance, in which mathematicians such as the creators of the calculus are seen as adventurous explorers, to newer stories of romantic and often "doomed" visionaries, such as Galois and Cantor.

Peter Galison uses the biographies of two pioneers, the flesh-and-blood mathematical physicist John Archibald Wheeler and the most famous pseudonymous mathematician in history, Nicolas Bourbaki, to show how their respective views of their craft, as well as the mathematics they created, were largely shaped by biography—actual in the first case, invented in the second. According to Galison, each mathematical argument tells a story, one not unrelated to its creator's own formative influences. Thus, Wheeler viewed mathematical arguments essentially as compound machines, a disposition quite possibly shaped by his early experiences as a boy growing on a Vermont farm. His childhood fascination with the intricate machinery around him led to his further, adolescent interest in watches, radios, and all sorts of technical contraptions and, eventually, to his thinking about mathematics as machine-like. Very different were the forms of mathematical argument proposed by the group of French mathematicians who published collectively as Nicolas Bourbaki. These were molded by and reflect the refined intellectual, bourgeois environment of Belle Époque Paris, ca. 1890–1914, in which the first members of Bourbaki grew up. In their book on the history of mathematics, the members of Bourbaki discard the until then dominant metaphor of mathematics as an edifice—a metaphor without which it would be hard to imagine any talk of "foundations of mathematics"—for one of a city whose great growth makes necessary the redesign and rebuilding of its central networks of roads and the creation of new, wide avenues capable of carrying the increased outward traffic. As Galison points out, this is an exact model of what had actually happened to Bourbaki's own city, Paris, in the mid-nineteenth century, when Georges-Eugène Haussmann demolished whole blocks of old, decrepit buildings in the city's center, as well as the labyrinths of alleys around them, to make way for his new, wide avenues.

A consciousness of the importance of metaphor to mathematical thinking is one of the insights at the heart of the new historiography of the field. Focusing on the question of whether belief can be said to play a part in mathematical thinking, in "Deductive Narrative and the Epistemological Function of Belief in Mathematics," Federica La Nave investigates one of the more epistemologically challenging breakthroughs in the history of mathematics by focusing on Bombelli's contribution to the creation of imaginary numbers. At a time when the square root of a negative quantity was considered an absurdity, the revolution in algebra spearheaded by a handful of Italian mathematicians would have been impossible without the quality of belief. La Nave's assumption that a new notion of it is necessary to understand Bombelli's creativity is supported by her close reading of his original texts. For though many of Bombelli's personal reasons for his belief in the existence of imaginary numbers—as, for example, the notion that algebra is essentially a calculating methodology, or that numbers might have a geometric representation—appear to us to be merely rational, in his writings we find a quality of affect transcending the sense of their obviousness, with which the axioms and the logical, certainty-preserving operations of deductive mathematics were traditionally approached.

The notion of belief leads to that of theology. In "Hilbert on Theology and Its Discontents: The Origin Myth of Modern Mathematics," Colin McLarty examines an instance of the direct attribution of the term to a mathematical theorem, Paul Gordan's famous comment on David Hilbert's extension to an arbitrary number of dimensions of his finite basis theorem: "This is not mathematics, this is theology!" After providing the background to the various interpretations that the comment has received, McLarty focuses on Gordan's one and only doctoral student, Emmy Noether, a mathematician who played a most important role in the creation of abstract algebra. In McLarty's treatment, the concept of theology becomes crucial to understanding the development of abstraction at the heart of Noether's thought. Going further, his essay discusses the little tale about Gordan as an unusually clear example of the deliberate use of narrative in mathematics, with all that this entails for understanding the history of mathematics.

One of the main characteristics of this volume is its multidisciplinary nature, seen also in the tendency of the practitioners of one field to temporarily abandon their own intellectual habits and attempt excursions into neighboring fields. This is perhaps most apparent in the essays by mathematicians writing about their peers' lives with a strong sense of the cognitive attributes of storytelling, attributes thought by many to set it at the antipodes of mathematical thinking. Interestingly, two of the essays in this book focus on the conventionally highly unmathematical concept of dream.

In "Do Androids Prove Theorems in Their Sleep?," Michael Harris chooses as the springboard for his discussion of dreams the decision by Robert Thomason to add as co-author of an important paper his deceased friend Tom Trobaugh, a non-mathematician. The reason Thomason gave was that Trobaugh, who had committed suicide a few months earlier, appeared to Thomason in a dream and, by uttering a single (wrong) mathematical statement, provided Thomason with the key step that allowed him to complete a particularly difficult proof. After a close reading of Thomason's description of the dream and an explanation of the underlying argument, Harris observes that a proof has an essentially narrative structure, then ventures into a general exploration of the similarities of mathematical proofs and works of fiction. In Harris's analysis, the issue of the role of dreams in mathematical research leads to an examination of the differences between intuition and formalism, or the differences between many actual proofs and the idealized notion of a completely formal one. The idealized proof is the central paradigm for theorem-proving computer programs, or "androids." Harris concludes his essay with some thoughts on the future possibility of collaborative proofs in which human and machine work together.

The role of dreams in mathematics is also the subject of one of the two editors' contributions. Barry Mazur begins his "Visions, Dreams and Mathematics" with an attempt—an attempt also undertaken by other contributors—to discuss a possible taxonomy of mathematical narratives, which he distinguishes according to the following classes: "origin stories," which can be thought of as coming from the non-mathematical world in the form of actual problems inspiring an investigation; "purpose stories," which again give non-mathematical reasons as the ultimate aim of a certain piece of mathematics; and stories he calls "raisins in the pudding," or purely ornamental and, in this sense, unnecessary. His own particular interest is in a fourth kind of story, which describes a vision of a grand project, or *dream*—this is a different use of the word from Harris's. The inspiration for Mazur's discussion is a comment made by a mathematician concerning a colleague: "He is an extraordinary mathematician, but he has no dreams." Mazur investigates what it means for a mathematician to have a dream by focusing on what Leopold Kronecker called his *liebster Jugendtraum*, the "beloved dream of (his) youth," which some mathematicians know as Hilbert's 12th problem. A great mathematical dream engenders in the mathematician a responsibility to follow it wherever it may lead, but it may extend further, even beyond the original dreamer's life. What makes Kronecker's *liebster Jugendtraum* dream so powerful is that its seeds lie in mathematics created before his times, specifically in an idea of Gauss, and it continues to motivate mathematicians long after the completion of his work. The exploration of this great mathematical dream propels Mazur in a discussion of basic concepts in the epistemology of mathematics, such as the difference between explicit and implicit statements.

The authors of the next two essays, Timothy Gowers and Bernard Teissier, are also mathematicians. However, unlike Harris and Mazur, who deal with concrete historical narratives of mathematics, Gowers and Teissier attempt a more general investigation of some generic similarities of mathematics and narrative. Gowers, in "Vividness in Mathematics and Narrative," uses the term *narrative* to refer to the most eminent subset of the set of all narratives: literary fiction. His discussion is focused on stylistics, and more specifically on a particular aspect of literary style, vividness, which is also a prime characteristic of a good presentation of mathematical ideas. Whether approaching a literary or mathematical text, the reader is pre-equipped with a vast web of ideas and images derived from previous experience. In the writings of great stylists in both fields, a small trigger in the text may be all that is needed to push a complex selection of such ideas and images to the front of the mind. Gowers gives examples of the wonderful vividness of great literary writing; he argues that when working through a totally analogous process, exactly the same response can be created in certain mathematical texts.

Bernard Teissier, in asking "Why are stories and proofs interesting?," looks at the interrelation of mathematics and narrative, centering on the notion of a clue. A strong motivation in mathematical research is the desire to uncover hidden facts or structures. In this metaphorical treasure hunt, the brain reads certain signs as more important than others, just as it would in following an adventure story or mystery. However, just as in novels, clues in mathematics can be misleading. No mathematician ever approaches a problem without the prejudices of his or her training, expertise, and likes and dislikes. Grothendieck has said that mathematical investigators ought to be like children and follow the leads without any preconceived idea—this innocence is, of course, easier wished for than achieved for knowledge of generic structures and forms is ingrained in a trained mathematician's brain, and it is this knowledge that, to a large extent, guides his or her search through a maze of possibilities. In this sense, a mathematician can no more be totally innocent than can a character in a story. But whereas in a realistic story a character's perception and interpretation of clues is based on knowledge of the real world, in works such as Lewis Carroll's *Alice in Wonderland* or James Joyce's *Ulysses*, much of the knowledge is internal to the texts and manipulated in their endless games with language. Teissier argues that some of the more formal criteria for clue hunting are characteristic of the appreciation of a mathematical landscape: the mathematician's thinking is informed both by intuitions that are essentially cognitive universals and by a sophistication acquired through experience with the formal games of mathematics.

Though mathematics is traditionally considered the logical discipline par excellence, in "Narrative and the Rationality of Mathematical Practice" David Corfield proposes that to be fully rational, mathematicians must embrace narrative as a basic tool for understanding the nature of their discipline and research. Starting from philosopher Alisdair MacIntyre's discussion of a tradition-constituted enquiry, Corfield argues for the partial validity of a pre-Enlightenment epistemology of mathematics as a craft whose advance is made possible only through a certain discipleship. Rather than view mathematical progress as the addition of newer pieces to an ever-growing jigsaw of abstract knowledge consisting of conjectures and theorems, or "mathematics as commodity," Corfield sides with the mathematicians Connes, Grothendieck, Thurston, and others who promote a vision of mathematics as understanding, an understanding that is inseparable from the narratives the discipline develops of its own progress. A narrative understanding of mathematical progress becomes a necessary part of a practice that fully accepts the reality and the importance of historically defined standards.

The contribution of Corfield uses the term *narrative* in a sense that is becoming increasingly prevalent in the human and social sciences: as a serial structuring device, usually in chronological time, which may or may not also possess some of the classical attributes of storytelling such as plot, characters, atmosphere, and so on. This sense opens a path to a more general investigation—which plays a central part, in varying guises, in many of the essays in this volume—of the underlying similarities between the cognitive practices of mathematics and of narrative. The next three essays attempt to better understand these similarities, through structural and formal comparisons, or use them for the better understanding of the cognitive history of mathematics.

The essay of the other editor, Apostolos Doxiadis, titled "A Streetcar Named (among Other Things) Proof: From Storytelling to Geometry, via Poetry and Rhetoric," also works with the notion of narrative as sequential representation, a notion that is more general than "story." Based on cognitive science and narratology, as well as the study of the rhetorical and poetic storytelling traditions of archaic and classical Greece, Doxiadis gives an account of the birth of deductive mathematics partly as a passage from one mode of thought (narrative) to another (logic). Rather than attempting to solve specific problems of relation or measurement, as their predecessors in Mesopotamia and Egypt did, classical Greek mathematicians constructed general propositions that they attempted to establish beyond any possible doubt. Doxiadis works in the tradition of Jean-Pierre Vernant and G. E. R. Lloyd, who see the new, participatory institutions of the late archaic and classical polis as a crucial factor facilitating the emergence of rationality in Greece. More particularly, he examines the culturally determined overlap of geometric thinking with the practices that developed in the courtrooms and assemblies of the classical Greek polis, under the pressing new civic need for deciding between conflicting views of reality. To better understand the interrelations of apodeictic methods in forensic rhetoric and mathematics, Doxiadis attempts to trace the roots of the former in techniques developed in archaic Greece, both in quotidian narrative and poetic storytelling.

One of the greatest stumbling blocks to perceiving mathematics from the point of view of narrative is the traditional conception of the field as dealing exclusively with timeless, absolute—and thus atemporal—truths, a conception going back to Plato's notion of mathematical truth. In his "Mathematics and Narrative: An Aristotelian Perspective," G. E. R. Lloyd shows that the conception of mathematics as atemporal was challenged, soon after Plato defended it, by Aristotle, who held that mathematical proofs are produced by an actualization (*energeia*). According to this view, which is in harmony with actual Greek mathematical practice, geometric relations exist in diagrams only as potential, being actualized in the process of proof. With his argument, Lloyd provides essential background to many of the essays in this book, if not to its very existence: narrative is only possible in time, and the expulsion of the temporal dimension in the Platonic view of mathematics could be argued to make any notion of the relationship of mathematics and narrative an oxymoron. By explaining the alternative Aristotelian view, Lloyd essentially legitimizes the whole range of our inquiry. For although, as he points out, actual chronology is not relevant to mathematical arguments, sequentiality—which is often time dependent—is. It is precisely on this ground that the two processes, of telling a story and of constructing a proof, often converge.

In "Adventures of the Diagonal," Arkady Plotnitsky sees the passage that occurred in the nineteenth century to what he calls non-Euclidean mathematics—a more encompassing category than non-Euclidean geometries —as having precise analogies with new kinds of mathematical narratives. The older, "Euclidean" mathematics, as well as the classical physics that it went hand in hand with, was related to narratives that depended on motion and measurement. By contrast, Plotnitsky sees non-Euclidean mathematics as partly relying on new cognitive paradigms of what the world is like, paradigms that include as central the abandonment of the very notion of the “object” as something that can be either discovered or constructed. Plotnitsky sees the first instance of what much later comes into full bloom as a non-Euclidean epistemology in the discovery by classical Greek mathematicians of the concept of "incommensurability," or the irrationality of certain numbers, such as the square root of 2. From the paradigm developed from such, somehow immaterial—because nonconstructible—entities, Plotnitsky attempts to trace a notion of narrative lacking the Kantian concept of the object at its center. He traces an account of this idea all the way from Greek incommensurability to the most advanced concepts of modern mathematics, such as Grothendieck’s topos theory or the Langlands program. Unlike the object-based, Euclidean narratives that for many centuries guided the language, the perceptions, and the concepts of mathematical understanding, non-Euclidean mathematics works through narratives that are closer to complex and tragic—in the sense of dialogic or ironic—views of reality.

From our earliest discussions, we thought of this volume as an opportunity for a two-way interaction between mathematics and narrative. The essays introduced to this point speak, in one way or the other, either of narratives of mathematics, or the structural and historical affinities of the two practices. For our overview of the interplay of mathematics and narrative to be more complete, however, we also need to travel in another direction: from mathematics *to* narrative. To do this, we asked three scholars from coming from narratology and literary studies to discuss, from the point of view of their own investigations, the influence of mathematical-type thinking on the study of narrative.

The first of these three essays, can also serve as a bridge, from mathematical to narratological territory. In "Mathematics and Narrative: a Narratological Perspective," Uri Margolin works in the same general area as Gowers and Teissier, that of the overlap of mathematics and literature, but looks at it from the other side of the hill. His particular interest is in the ways in which we can speak of mathematics *in* literature, as for example the cases of literary narratives with mathematicians as heroes; narratives in which plots are presented as a mathematical object, like a cryptogram; texts with a formal mathematical structure overriding the more usual, mimetic function of literature, as in the experimental works of the Oulipo group; or works of fiction, like some stories of Jorge Luis Borges, in which a mathematical notion, such as infinity or branching, functions as a key topos. The greater part of Margolin's essay, however, is given over to a detailed typology at a finer level, and more particularly to the investigation of the structural similarities and differences between how mathematical texts and narratives treat the creation of imaginary worlds, and the criteria of truth, levels and hierarchies of representation involved in this process. A large part of this analysis is based on the concepts of information and choice, as well as related structures of games and searches in both mathematics and narrative, building on ideas presented in John Allen Paulos's *Once upon a Number* (1999).

In his essay, "Formal Models in Narrative Analysis," David Herman provides a thoughtful overview of the existing formal models of narrative, whose creation was one of the main driving forces behind the development of narratology, a field that is undergoing a renaissance, chiefly because of its interaction with cognitive studies. Herman surveys some of the motivations, benefits, and problems of the models proposed by scholars working in a variety of fields, drawing on mathematical understandings of the concept of model to reflect on the nature of the theory of narrative. His contribution has both a diachronic, genealogical scope and a synchronic, diagnostic one. On the one hand, he explores the historical background of some instances of the confluence of the formal study of narrative and mathematics, such as the use of permutation groups, as well as the synergy between mathematically based theories of structural linguistics and early work on story grammars. On the other, he compares models developed by students of narrative, placing these in larger conceptual frameworks, each one determined by certain assumptions about what stories are and how best to study them.

The last essay, by Jan Christian Meister, presents in detail the logic of one particular formal model. In "Tales of Contingency, Contingencies of Telling: Toward an Algorithm of Narrative Subjectivity," Meister shows how the narrating voice, the actions and thoughts of the characters, and readers' cognitive and emotional responses always bear traces of individuality, an individuality that is almost impossible to formalize. Sometimes it may be possible to adequately describe, and perhaps even explain, the behavioral logic of a narrator or character. Yet until a narrative is fully processed and the transformation of its words (or other symbolic material) into mental images has come to a close, with a coherent model of the referenced world firmly established, contingency reigns. In fact, a particular kind of unpredictability is a defining characteristic of the narrative mode. To approach formally the notion of narratorial subjectivity, Meister begins from the two ways in which theorists have tried to understand it. The first way is through perspective, which is a coding inside a narrative utterance signaling its stance with respect to what is narrated; the second is focalization, which sets the epistemological boundaries of what has been perceived or imagined in a narrative instance in order to be narrated. Meister investigates how these two concepts, perspective and focalization, can be formalized in the context of a theoretical story generator algorithm—"algorithm" already referring to mathematical concepts—and proposes ways in which mathematical tools may help in the modeling of narrative subjectivity.

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The authors of the works published in this volume met for a week during the summer of 2007 in Delphi, where they presented and discussed earlier drafts of their papers. This engagement led most of the contributors to rewrite their contributions, with the aim of making them parts of a more coherent whole. During the week of the meeting, each author was interviewed at length by another author about some of the issues discussed in his or her contribution. The recorded interviews were transcribed and assembled on the website www.thalesandfriends.org. We hope that this added resource will be useful to readers who wish to better understand some of the viewpoints expressed here.

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Apostolos Doxiadis
Barry Mazur

NOTE

1. Quoted in Gérard Genette, *Narrative Discourse: An Essay in Method*, trans. Jane Lewin (Ithaca, NY: Cornell University Press, 1980), 80–81