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Experience-Based Heterogeneity in Expectations and Monetary Policy

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Experience-Based Heterogeneity in Expectations and Monetary Policy

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Abstract

The present paper studies the effect of monetary policy on inflation and output within a New Keynesian model with Experience-Based Learning (EBL) that renders expectations heterogeneous across age groups. Under EBL, the age-distribution directly affects the composition of aggregate expectations which gives rise to a novel channel by which the demographic structure of an economy affects monetary policy: the Experience Channel. Relative to models with homogeneous and age-independent expectations, we show that EBL weakens the pass-through of monetary policy on aggregate demand. This affects monetary policy transmission in two ways. First, the impact response of inflation and output to monetary policy shocks is less pronounced since the response of expectations under EBL is muted. Second, the response of inflation is more persistent, since the interest rate sensitivity of aggregate demand is weaker. Moreover, EBL changes the classical monetary policy trade-off between output and inflation stabilisation under supply shocks relative to models with age-independent expectations. First, for a given inflation volatility, output is always less volatile due to the reduced sensitivity of expectations to monetary policy. Second, under EBL, the trade-off becomes more severe in ageing economies. We show that this effect is largely driven by the Experience Channel.

Keywords: Monetary Policy, Learning, Heterogeneous Expectations, Experience Effects, Demography

JEL Classification: D84, E32, E37, E52, E70

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1. Introduction

Private sector expectations are a key determinant for the implementation of monetary policy. In the standard New Keynesian model (e.g. Clarida et al., 1999), monetary policy is analysed within the rational expectations paradigm, which implies that economic agents hold homogeneous expectations. This implication, however, stands in contrast to the empirical evidence for cross-sectional heterogeneity in expectations (e.g. Branch, 2004; Pfajfar and Santoro, 2010). Indeed, extending the standard New Keynesian model by including heterogeneity in expectations alters the propagation mechanism of shocks (e.g. Branch and McGough, 2011) and the transmission of monetary policy (e.g. Branch and McGough, 2018; De Grauwe, 2011; Massaro, 2013). In this literature, heterogeneity in expectations results either from an ad-hoc division of the population into individuals with different forecasting models or from agents' discrete choice from a finite set of predictors (based on Brock and Hommes, 1997).

The present paper, instead, takes a different route and assumes that the heterogeneity in expectations results from different economic experiences individuals make over their lifetime, which is motivated by the empirical evidence of Malmendier and Nagel (2016). As a consequence, expectations are heterogeneous across age groups. We embed *Experience-Based Learning* (EBL) into a standard New Keynesian model and use the model to analyse the transmission of monetary policy and its stabilisation trade-off under supply shocks. To the best of our knowledge, the present paper is the first that studies monetary policy within a New Keynesian model with EBL.

Allowing expectations to differ across age groups provides a novel channel by which the age distribution affects the conduct of monetary policy. Given the heterogeneity in expectations across age groups, the demographic structure of an economy affects the composition of aggregate expectations which are essential for aggregate outcomes. We show that if this composition effect is disregarded, the effect of the age-distribution on the classical monetary policy trade-off between output and inflation stabilisation under supply shocks is understated.[1] Moreover, while a variation in the age-distribution has no meaningful impact of the transmission of monetary policy under age-independent expectations, the transmission of monetary policy on inflation under EBL is significantly more pronounced but less persistent in older economies.

Experience-Based Learning. The effect of life-time experiences on individual's expectation formation has been analysed empirically by Malmendier and Nagel (2016). They

[1] In the standard New Keynesian model, output and inflation move into opposite directions under supply side driven fluctuations (e.g. TFP shocks). In this case, monetary policy is unable to stabilise both output and inflation simultaneously with the nominal interest rate as the single instrument (e.g. Clarida et al., 1999).

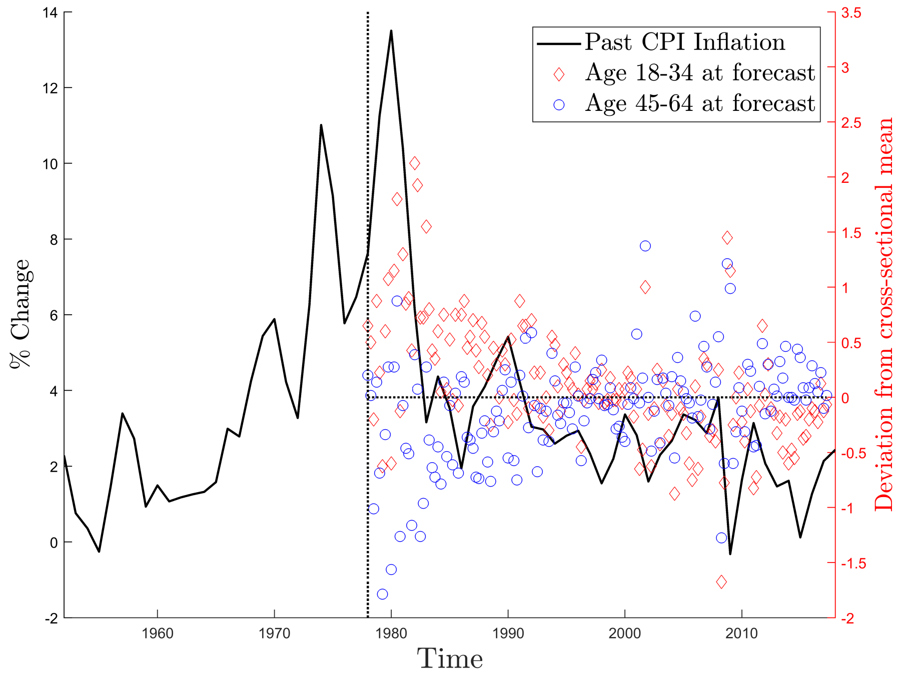

FIGURE 1: One-Year Inflation Expectations Across Cohorts

Notes: The black line denotes the annual percentage change of the seasonally-adjusted US-CPI taken from FRED. The markers are the deviations of the 4-quarter moving average of cohort inflation expectations from the cross-sectional mean in a given quarter in percentage points. We take a moving-average to concentrate on lower frequency variation. The data is taken from the Michigan Survey of Consumers (question: *Expected Change in Prices During the Next Year*) from 1978 onwards. Cohorts define persons of a certain age group at a specific point in time, so that no cohort is tracked over time.

find that differences in inflation expectations across age groups are to a large part driven by differences in their experienced inflation. In particular, they find that young cohorts' expectations are more sensitive to recent observations than those of old individuals.[2] Figure 1 illustrates this finding.[3] For a "young" and an "old" age group, we plot one-year inflation expectations as deviation from the cross-sectional mean (markers, in percentage points). The black line denotes the annual percentage change of the US-CPI (from the previous year, left y-axis). The heterogeneity in expectations across different age groups is particularly pronounced in the early 1980s. The comparison of the age-dependent inflation expectations with the annual percentage change of the US-CPI reveals that young individuals who observed only the high inflation rates during the 1980s tend to have higher inflation expectations than those individuals who also observed low inflation during the

[2] Experiences not only shape individuals' inflation expectations. Malmendier and Nagel (2011) show that experienced past asset returns affect individuals' risk taking behaviour and stock return expectations while Kaustia and Knüpfer (2008) show that experienced investment outcomes influence future investments. Moreover, Malmendier et al. (2020a) explore how inflation experiences of Federal Open Market Committee members affect their inflation expectations.

[3] This figure (partly) reproduces the one from Malmendier and Nagel (2016, p.55). To clarify the idea of EBL, we additionally plot the annual percentage change of the seasonally-adjusted US-CPI. This facilitates the comparison of expected inflation to experienced inflation.

1950s and 1960s. Importantly, experiences – not age by itself – matter which can be seen by the fact that in 2012 the pattern is reversed and the young age group expects lower inflation than the old one.
Given the heterogeneity in expectations across age, the demographic structure of an economy affects both the mean and persistence of average expectations. This provides a novel channel by which the age distribution affects the transmission of monetary policy.[4]

Incorporation of EBL into the New Keynesian Model. To study the effect of age-related heterogeneity in expectations on aggregate fluctuations and monetary policy, this paper embeds the empirical evidence of Malmendier and Nagel (2016) into a New Keynesian model with overlapping generations à la Blanchard (1985) and Yaari (1965).
Following the literature on statistical learning, households in our model no longer form rational expectations (RE, henceforth), but behave as econometricians who constantly revise the parameters of their forecasting rules on economic variables when forming expectations. We assume that agents form expectations on an economic variable based on a simple auto-regressive process of order one. We refer to the AR parameter as an agent's *perceived persistence*.
The point of departure from the statistical learning literature lies in the specification of the weight individuals attach to new information, which corresponds to the one identified by Malmendier and Nagel (2016). In particular, all agents – *irrespective of their age* – put more weight on recently observed data points rather than those observed early in life but ignore any data points that realised *prior* to their birth. Moreover, the weight individuals attach to new observations when updating their beliefs decreases in their age so that young individuals are more sensitive to new data points than older ones. We calibrate the New Keynesian model and use the empirical estimates of Malmendier and Nagel (2016) to specify the age-dependent weight individuals attach to new information. The calibrated model is then used for a quantitative analysis of the effect of monetary policy on inflation and output if experiences affect individuals expectation formation.

Properties of EBL. The model with EBL generates a quantitatively meaningful heterogeneity in expectations across different age groups. For a supply shock, calibrated as in Smets and Wouters (2007), expected inflation of young and old individuals differs up to ± 1.5 percentage points.
The source of expectations heterogeneity in our model is the heterogeneity in the persistence different cohorts attach to economic variables. When simulating the economy, we

[4]This becomes even more relevant given the projected ageing of the workforce in advanced economies. While for the period 1998-2028 the share of U.S. workers aged 25-54 years is projected to decline from 72% to 63%, the one of workers aged +55% may rise from 12% to 25% (according to data from Dubina et al., 2019). For other countries, even stronger shifts are projected.

find that young individuals' perceived persistence, on average, is lower and more volatile than the one of old individuals. This follows from the assumption that young agents put more weight on recent observations so that their beliefs are more sensitive to any new information. A stronger sensitivity with respect to new observations leads to higher dispersion in their perceived persistence. Furthermore, the low amount of data on which young agents base the parameter estimates of their forecasting rules results in *lower* persistence in young agents' beliefs, on average.[5]

Pass-through of Monetary Policy. We find that in the model with EBL, the stabilising power of simple monetary policy rules relative to a model with RE is weakened mainly due to a lower pass-through on aggregate demand through expectations. There are two key driving forces that explain this finding. First, the pass-through is weaker under adaptive expectations *in general*. Intuitively, under RE, agents are forward-looking and internalise the Taylor rule's impact on the future path of real interest rates when forming expectations. These, in turn, affect current output via the Dynamic IS Curve. Under adaptive expectations, however, agents are backward-looking so that agents *fail* to project the effect of monetary policy on the future path of the real interest rate. Instead, movements in the nominal interest rate affect expectations only with a lag via its effect on current inflation and output, which are used to form expectations in the next period. Consequently, the effect of the *real* interest rate on aggregate demand is weakened relative to a model with RE.

Second, the lagged effect of monetary policy on aggregate expectations is lower the smaller the aggregate perceived persistence in the economy. For agents using EBL, the persistence attached to an economic variable determines how strong its expectation reacts to contemporaneous shocks.[6] In addition, a smaller perceived persistence reduces the impact of past monetary policy actions on current variables. We show that, on average, aggregate expectations are less persistent in a model with EBL relative to a model where we shut-off the experience effects and assume that all agents attach an age-independent constant weight on new information. Hence, the the pass-through of monetary policy on aggregate demand is weaker when taking into account experience effects.

[5] Hence, the empirical facts identified by Malmendier and Nagel (2016), most of all clearly discernible expectation heterogeneity, are not washed out in a DSGE-framework. Different channels might weaken their finding. First, monetary policy might react forcefully enough to stabilise inflation so that differences in experiences across age are too small to yield differences in aggregate expectations. Second, depending on their relative size and sensitivity to new observations, old agents might sufficiently stabilize aggregate expectations as to not allow significant swings in inflation.

[6] As shown in Slobodyan and Wouters (2012a), the aggregate perceived persistence in the economy affects the response of the endogenous variables in response to exogenous disturbances.

Stabilisation Trade–off. The lower pass-through of monetary policy under EBL compared to a model with RE affects the stabilising effect of simple policy rules. In a New Keynesian model, monetary policy faces a trade-off when aiming to simultaneously stabilise output and inflation under supply-side driven fluctuations (e.g. Galí, 2015). We show that this trade-off weakens in a model with EBL compared to the model with RE because output is always less volatile under EBL. This directly results from the lower pass-through on aggregate demand. The strength of this effect depends on the age-distribution. Under EBL, the age-distribution directly determines the composition of aggregate expectations. An increase in the mass of young agents, which attach low persistence to economic variables, further reduces aggregate perceived persistence in the model. In consequence, the pass-through gets even weaker which further alleviates the trade-off. This provides a novel channel by which the age-distribution affects monetary policy, which we call the *Experience Channel*. We show that the effect of a variation in the age-distribution on the strength of the trade-off is understated if we shut off the Experience Channel and assume that all agents put a constant weight on any new information.

Transmission of Monetary Policy Shocks. Comparing the impulse responses to a monetary policy shock, we show that the response of inflation is less pronounced, on impact, but displays a higher persistence in the subsequent reversion to the steady state. The lower impact response results from the fact that expectations are influenced by shocks to a smaller extent because of a lower perceived persistence. The more persistent response of inflation results from the reduced ability of monetary policy to influence aggregate demand. Moreover, while the demographic structure has a negligible effect on the shock transmission in the model with RE, we find substantial differences in the response of inflation under EBL. In the latter model, inflation responds stronger and more persistently in an "old" economy where the perceived persistence of inflation is, on average, higher than in a "young" economy. Hence, the Experience Channel matters for the transmission of monetary policy shocks.

Related Literature. Our work is related to several strands of literature. First, we relate to the adaptive learning literature that studies monetary policy within a New Keynesian model. In such a framework, the "Taylor Principle" is a necessary but not a sufficient condition for stability under learning (Bullard and Mitra, 2002). Evans and Honkapohja (2006) and Preston (2006) show that monetary policy can reduce problems arising from instability under learning by taking into account how bounded rational agents form expectations. Eusepi and Preston (2010) and Orphanides and Williams (2005) argue that central bank communication about the rules it uses to stabilise inflation and output reduces the uncertainty in forecasting models of private agents which enhances stability.[7] In comparison to this literature, we assume that individuals are agnostic about the minimum state variable representation of the economy and instead form expectations based on simple autoregressive forecasting models. We further assume that the parameters of the forecasting model are recursively updated and differ across age group so that expectations are heterogeneous across generations. Moreover, our analysis abstracts from a discussion of determinacy or optimal policy and instead focuses on the transmission of monetary policy and its stabilisation trade-off under supply shocks under different demographic structures.

Second, we contribute to the literature that analyses monetary policy within a New Keynesian model with heterogeneous expectations. Branch and McGough (2009) show that once the assumption of homogeneous expectations is relaxed, the Taylor principle is not sufficient for monetary policy to guarantee determinacy of the equilibrium. Gasteiger (2014) uses his framework to study optimal monetary policy and finds that local determinacy prevails if monetary policy implements the RE equilibrium with an expectations-based reaction function that accounts for the heterogeneity in expectations. Massaro (2013) analyses monetary policy within a New Keynesian model with heterogeneity in long-horizon expectations and generalises the results from Branch and McGough (2009). In contrast to this literature, we assume that the heterogeneity in expectations results from different experiences individuals make over their lifetime which renders expectations heterogeneous across age groups. Further, in our model, the demographic structure affects the composition of aggregate expectations which provides a novel channel by which the demographic structure affects the conduct of monetary policy.

Third, experience-based expectation heterogeneity has already been analysed in theoretical models. However, most focus on asset pricing in partial equilibrium (instead of monetary policy in general equilibrium, like us), as Collin-Dufresne et al. (2016), Ehling et al. (2018), Malmendier et al. (2020b), Nagel and Xu (2019), and Schraeder (2015). The only model using EBL in a GE framework that we are aware of is Acedański (2017) who explore its implication on the wealth distribution. To the best of our knowledge, we are the first who use a New Keynesian model to explore monetary policy implications that stem from demography-induced effects of EBL.

Last, we contribute to the literature that studies the link between demographic changes and monetary policy. There is broad agreement that in the long-term longevity and declining birth rates contribute to a reduction in the real interest rate (e.g. Aksoy et al., 2019; Eggertsson et al., 2019; Kara and von Thadden, 2016), which carries the risk of a binding zero-lower bound. Wong (2019) considers the transmission of monetary policy through the refinancing channel of mortgages and finds that young households react

[7] See Eusepi and Preston (2018) for a more detailed survey of the literature.

stronger to a monetary policy shock. In contrast, Berg et al. (2020) argue that old agents are, in the aggregate, more responsive since they hold more interest-rate sensitive wealth than young agents. Given an ageing society both studies deliver different conclusions on the efficiency of monetary policy transmission. Our work studies a hitherto unconsidered channel of how shifts in demography affect the transmission of monetary policy through experience effects. Our results suggest that in an ageing society the transmission of monetary policy is stronger, yet the stabilisation trade-off between output and inflation is more severe.

Structure. We proceed as follows. Section 2 presents our NK-model with an OLG-structure. In section 3 we explain the details of EBL. Our results concerning EBL properties, dynamic responses, and monetary policy are discussed in section 4. Finally, section 5 concludes and gives an outlook on further work.

2. The Model

In the present section, we consider a New Keynesian framework. However, we deviate from the standard model (e.g. Galí, 2015, Ch.3) by assuming that households form expectations based on their life-time experiences. Conceptually, we follow the statistical learning (SL, henceforth) literature that assumes agents to have a simplified model of the economy in mind by which they form beliefs about future outcomes not knowing the actual law of motion of the economy. Agents constantly revise their beliefs as new observations become available to them. Since we allow this revision of beliefs to depend on an agent's age, we assume households to face a constant probability of death á la Blanchard (1985) and Yaari (1965).[8] Their saving decision involves the formation of inflation and output expectations, which depend on their age and life-time experiences. In our model this is the only source of heterogeneity across agents. Taken by itself, heterogeneous expectations lead to different saving decisions across agents. However, similar to other literature using boundedly rational agents (e.g. Mankiw and Reis, 2007, Adam et al., 2016 and Ehling et al., 2018) we abstract from the wealth distribution as an additional state to better focus on the effects stemming from EBL via the expectation operator. To achieve this we employ the approach of Branch and McGough (2009) as pointed out in appendix A.3.2. Households own intermediate good producers that use labour to provide an input to a competitive final good producer whose output households consume. Monetary policy influences the bond rate according to a Taylor (1993)-rule.

[8]We exclude the possibility of individuals to retire after their working life to keep the model simple.

2.1. Households

Households provide labour, form expectations according to EBL and own intermediate good firms. At each point in time, the mass of households is constant and normalised to one. They face an age-independent probability, $\omega \in [0,1]$, of surviving into the following period. In turn, at the beginning of each period a share of $1-\omega$ households deceases and is replaced by new-born households of equal mass. Consequently, the mass of a cohort born in period k at time $t \geq k$ is given by $(1-\omega)\omega^{t-k}$.

A household born in period k maximises the discounted sum of life-time utility:

$$\tilde{E}_t^k \sum_{j=0}^{\infty} (\beta\omega)^j \, u(c_{t+j|k}, l_{t+j|k}) \ , \tag{1}$$

subject to the sequence of period budget constraints:

$$p_t c_{t|k} + b_{t|k} = r_{t-1}(b_{t-1|k} + z_{t|k}) + p_t w_t l_{t|k} + \mathcal{D}_{t|k} \ , \tag{2}$$

where β denotes the discount factor, p_t the price level and $\tilde{E}_t^k$ denotes the subjective expectations operator, which potentially differs across cohorts k and is specified below. Households from cohort k receive labour income which is the product of nominal hourly wages, $p_t w_t$, and working hours in cohort k, $l_{t|k}$. They invest in private one-period nominal bonds, $b_{t|k}$, which pay nominal interest rate r_t, known as of t, tomorrow. We assume each household owns equal shares in every firm so that nominal dividends are equal across cohorts, i.e. $\mathcal{D}_{t|k} = \mathcal{D}_t$.

The time of death is uncertain and households may die with wealth. To avoid the inefficiency of accidental bequests we follow Blanchard (1985) and introduce insurance companies that pay bond shares as annuity payments $z_{t|k}$ and that receive all assets at the time of death. Profits for a particular company contracting with cohort k are:

$$\pi_t^I = (1-\omega)\, b_{t-1|k} - \omega z_{t|k} \ .$$

Due to free entry insurers make zero-profits so that $z_{t|k} = \frac{1-\omega}{\omega} b_{t-1|k}$. The above sequence of period budget constraints is supplemented with a solvency condition of the form

$$\lim_{T\to\infty} \tilde{E}_t^k \left\{ \mathcal{R}_{t,T} b_{T|k} \right\} = 0 \ , \tag{3}$$

where $\mathcal{R}_{t,T} = (\prod_{s=t+1}^{T} r_s)^{-1}$. We assume the following form of the felicity function:

$$u(c_{t|k}, l_{t|k}) = \ln\left(c_{t|k}\right) + \psi_n \ln\left(1 - l_{t|k}\right) \ ,$$

where ψ_n is a utility weight. Then, maximising (1) subject to (2) yields the optimal consumption/saving decision:[9]

$$1 = \tilde{E}_t^k \left\{ \beta \frac{p_t c_{t|k}}{p_{t+1} c_{t+1|k}} r_t \right\} . \tag{4}$$

Equation (4) denotes the household's Euler equation. While households of all ages face the same nominal interest rate, they have different expectations of the real rate. Hence, a household expecting a high future return, saves more and values future consumption more strongly than a household whose past experiences make her believe in dismal real future returns.[10] Notwithstanding that age-related heterogeneity in expectations implies differences in cohort wealth, we aggregate the economy without considering the wealth distribution as an additional state variable as outlined in Section 2.3.

Furthermore, we follow Evans and Honkapohja (2012) and Slobodyan and Wouters (2012a) that require near-rational agents to forecast variables only one period ahead (e.g. of variables in their Euler equation so that the approach is called *Euler equation learning*). Hence, the Euler equation for the current period gives households decisions as a function of the expected state of the economy tomorrow only.[11]

We also derive the labour supply of a household from cohort k that, via different consumption choices among cohorts, is cohort specific:

$$\psi_n \frac{c_{t|k}}{(1 - l_{t|k})} = w_t . \tag{5}$$

2.2. Firms

There are two types of firms. Final good firms use intermediate inputs to provide an aggregate consumption good. Intermediate good firms are owned by households and operate on a monopolistically competitive market. The choice to set up firms as in the usual NK-model serves to make our departure from the standard case minimal.

2.2.1. Final Good Firm

The aggregate consumption good in the economy, y_t, is produced by a perfectly competitive firm which is aggregating intermediate goods $i \in [0, 1]$ produced by intermediate

[9] Derivations are delegated to Appendix A.

[10] Malmendier and Nagel (2011) provide empirical evidence that experience effects in individuals' expectation formation contribute to differences in their savings decision.

[11] For further details on the implication of Euler-equation learning on agents behaviour see Appendix A.3.2 and Evans et al. (2013). Other approaches assume agents form forecasts on longer horizons, e.g. Preston (2005).

firms according to the technology:

$$y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} , \tag{6}$$

where $\varepsilon > 0$ is the elasticity of substitution among the intermediate goods, $y_{i,t}$. The final good firm chooses the quantities of intermediate goods to maximise its profits. The demand for intermediate good i is given by:

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t} \right)^{-\varepsilon} y_t \ , \tag{7}$$

where $p_{i,t}$ denotes the price at which the intermediate good firm i sells the input to final good producers (see appendix A.2 for derivations).

2.2.2. Intermediate Good Firms

Each household alive in period t owns an equal share in each intermediate good firm $i \in [0, 1]$ that produces a differentiated good on a monopolistically competitive market. Since all households are involved in firms to an equal degree, the latter have average expectations as detailed below. We assume that the share of a deceasing household is transmitted to a new-born one instantaneously. Production of good follows the technology:

$$y_{i,t} = x_t \, l_{i,t}^{\alpha} \, , \tag{8}$$

where $l_{i,t}$ denotes labour demand of firm i and x_t represents productivity per working hour and $0 < \alpha \leq 1$. We assume that TFP (asymptotically) follows a log-normal process:

$$x_t = x^* \exp\left(\epsilon_t^x\right) \tag{9}$$

$$\epsilon_t^x = \rho_x \epsilon_{t-1}^x + \nu_t^x \quad \text{with} \quad \nu_t^x \overset{iid}{\sim} \left(0, \sigma_x^2\right) \ . \tag{10}$$

Intermediate firm i sells its good at price $p_{i,t}$ but, when changing its price, pays quadratic nominal price adjustment costs à la Rotemberg (1982).[12] Hence, the firm faces an intertemporal problem that stems from the effect of $p_{i,t}$ on future price adjustment costs. The costs of changing prices are proportional to the nominal value of aggregate production:

$$\frac{\phi}{2} \left(\frac{p_{i,t}}{p_{i,t-1}} - 1 \right)^2 p_t y_t,$$

[12] Note that by assuming firms to face Rotemberg (1982) adjustment cost, we avoid that households face idiosyncratic labour income risk that would be present if firms faced Calvo-type price-setting rigidities. The absence of idiosyncratic risk precludes the need to include an additional insurance mechanism that guarantees perfect risk sharing *within* each group of household with expectations of type k (e.g. Mankiw and Reis, 2007).

where ϕ measures the degree of nominal rigidity. The adjustment cost increase in the scale of price changes and in the size of economic activity. Current real period profits, $d_{i,t} = \frac{\mathcal{D}_{i,t}}{p_t}$, of firm i are given by:

$$d_{i,t} = \frac{p_{i,t}}{p_t} y_{i,t} - w_t l_{i,t} - \frac{\phi}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 y_t$$

Taking aggregate prices as given, firm i chooses $p_{i,t}$ and $y_{i,t}$ to maximise discounted profits:

$$\max_{p_{i,t+j}, y_{i,t+j}} \bar{E}_t \sum_{j=0}^{\infty} \omega^j Q_{t,t+j}\ d_{i,t+j}\ ,$$

subject to the demand schedule of final good firms (7). The expectation operator $\bar{E}_t \mathbf{z}_{t+1} \equiv (1-\omega)\sum_{k=-\infty}^{t} \omega^{t-k} \tilde{E}_t^k \mathbf{z}_{t+1}$ denotes the aggregated expectations across single households for a generic variable $\mathbf{z}$ and is a size-weighted sum of cohort expectations. Note that the generational structure matters for aggregating the decisions of the households of different age and especially when aggregating the expectations of differently aged households. Since households hold equal shares in every firm, firms use a weighted average of household expectations.[13] Further, $Q_{t,t+j} \equiv \beta^j \frac{p_t c_t}{p_{t+j} c_{t+j}}$, where $c_t = (1-\omega)\sum_{k=-\infty}^{t} \omega^{t-k} c_{t|k}$, is the aggregate SDF of households.

2.2.3. Monetary Policy

The nominal interest rate on bonds is determined by a monetary policy authority that sets it according to a feedback rule:

$$r_t = \bar{r}\left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi}\left(\frac{y_t}{y}\right)^{\varphi_y} \exp\left(\epsilon_t^m\right) \tag{11}$$

$$\epsilon_t^m = \rho_m \epsilon_{t-1}^m + \nu_t^m \quad \text{with} \quad \nu_t^m \overset{iid}{\sim} \left(0, \sigma_m^2\right)\ , \tag{12}$$

where $\bar{r}$, π and y are the steady state interest rate, aggregate inflation and output, respectively. The parameters φ denote the feedback coefficients that determine the sensitivity to inflation and output deviations from steady state. Last, ϵ_t^m serves as monetary policy shock and evolves according to an AR(1)-process. We specify the monetary policy authority to use current inflation (opposed to its expectation), to avoid taking a stance on which type of expectations the monetary policy-maker has. Moreover, the central bank reacts to deviations of output and not of the output gap. Although both measure distinct concepts, in the linearised form of the model they differ only by TFP. Including the output gap complicates the model in the sense that the natural rate of interest appears in the

[13] Coibion and Gorodnichenko (2015) find firms' inflation expectations to be better captured by household survey data than by professional forecasters.

Dynamic IS Curve as the expected change of TFP, which agents would form using EBL. We abstract from another channel of EBL for the sake of more precise statements.

2.3. Equilibrium

Labour market equilibrium. As all intermediate firms produce with the same technology, equilibrium labour demand is symmetric. Aggregate working hours follow as:

$$l_t^d = \int_0^1 l_{i,t} di = \int_0^1 \left(\frac{y_{i,t}}{x_t}\right)^{\frac{1}{\alpha}} di = \left(\frac{y_t}{x_t}\right)^{\frac{1}{\alpha}} \Delta_t^p = l_t^s = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} l_{t|k} \; , \tag{13}$$

where $\Delta_t^p \equiv \int_0^1 \left(\frac{p_{i,t}}{p_t}\right)^{-\frac{\varepsilon}{\alpha}} di$ is an index of relative price distortions. Under Rotemberg pricing the symmetric price equilibrium implies $\Delta_t^p = 1$.

Goods market equilibrium. An equilibrium on the aggregate goods market requires that the total number of goods produced, y_t, equals the total amount of goods demanded, taking into account the dead-weight loss due to repricing cost:

$$y_t = c_t + \frac{\phi}{2}(\pi_t - 1)^2 y_t \; , \tag{14}$$

where $\pi_t = \frac{p_t}{p_{t-1}}$ denotes aggregate (gross) inflation.

Bond market equilibrium. Private bonds are in zero net supply, that is:

$$(1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} b_{t|k} = 0 \; . \tag{15}$$

New Keynesian Phillips Curve. Using the FOC on prices of intermediate good firms and symmetry, one can derive:[14]

$$(\pi_t - 1)\,\pi_t = \omega \bar{E}_t \left[Q_{t,t+1} \frac{y_{t+1}\pi_{t+1}}{y_t} (\pi_{t+1} - 1)\,\pi_{t+1} \right] + \frac{\varepsilon(1+\eta_l)}{\phi} \left(\mathrm{mc}_t^r - \mu\right) \; , \tag{16}$$

where mc_t^r are the real marginal cost, $\mu \equiv \frac{\varepsilon - 1}{\varepsilon}$ denotes the steady state markup and $\eta_l = \frac{l}{1-l}$ denotes the stationary labour-leisure share. Note that *aggregate* inflation expectations, π_{t+1}, affect π_t. According to (16), optimal price setting requires inflation to be a function of current real marginal cost and expected future inflation.

Linearised Equilibrium Conditions. In the current formulation expectation heterogeneity matters for households' Euler equations and the Philips Curve. To arrive

[14] Derivation, see appendix A.2.

at an aggregated dynamic IS-curve, we follow the literature and rely on Branch and McGough (2009).[15] First, we adopt their assumption on higher-order beliefs: household i's expectation about what another household k expects, is its own expectation: $\tilde{E}^i_t \tilde{E}^k_t \mathbf{z}_{t+1} = \tilde{E}^i_t \mathbf{z}_{t+1}$, $i \neq k$ for some generic variable $\mathbf{z}$, which reduces the complexity imposed on the model considerably.[16] Second, we assume agents expect to hold the same wealth in the limit $t \to \infty$. For each agent i consumption then equals the long-run consumption: $\tilde{E}^i_t (\hat{c}_\infty - \hat{c}^i_\infty) = 0$. This assumption prevents the wealth distribution from appearing in the aggregated IS-curve, which is beyond the scope of the paper.[17] After linearisation and aggregation the model can be summarised as system of 5 equations and 5 variables $\{\hat{y}_t, \hat{\pi}_t, \hat{r}_t, \hat{\epsilon}^x_t, \hat{\epsilon}^m_t\}_{t=0}^{\infty}$.

$$\hat{y}_t = \bar{E}_t \hat{y}_{t+1} - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1}\right) \tag{17a}$$

$$\hat{\pi}_t = \beta\omega \bar{E}_t \hat{\pi}_{t+1} + \kappa \left(\hat{y}_t - x^* \hat{\epsilon}^x_t\right) \tag{17b}$$

$$\hat{r}_t = \varphi_\pi \hat{\pi}_t + \varphi_y \hat{y}_t + \hat{\epsilon}^m_t \tag{17c}$$

$$\hat{\epsilon}^x_t = \rho_x \hat{\epsilon}^x_{t-1} + \nu^x_t \tag{17d}$$

$$\hat{\epsilon}^m_t = \rho_m \hat{\epsilon}^m_{t-1} + \nu^m_t \ . \tag{17e}$$

where $\kappa \equiv \frac{\varepsilon(1+\eta_l)}{\phi}$ denotes the slope of the NK Phillips curve.[18] To solve the model, we need to specify how agents form expectations.

3. Expectation Formation

In this section, we specify how different cohorts form expectations on inflation based on Malmendier and Nagel (2016). We first explain how a single cohort forms expectations and then turn to why experience effects play an important role. Last, we highlight how the EBL-approach differs from constant-gain learning (CGL, henceforth), which is among the most popular learning approaches.

3.1. Learning

Consider the model that agents use to form expectations. A large part of the literature assumes that agents know the true state-space representation (hence, they know the rele-

[15]See Appendix A.3.2. Aggregation allows to obtain a formulation close to the standard NK case.

[16]Having assumed that agents do not know the structure of the economy, imposing that they can not foresee how others form expectations is consistent. For approaches explicitly taking into account higher order beliefs consider Angeletos et al. (2018) or Farhi and Werning (2019).

[17]This can be seen when aggregating the single linearised Euler equations as we do in Appendix A.3.2. Essentially, the assumption cancels the differences in expected consumption at $t \to \infty$ that occur when aggregating expected consumption for households that have heterogeneous expectations and solve a dynamic problem.

[18]The PLMs' updating equations (20a) and (20b) are also part of the model.

vant variables for the economy's evolution) but have to learn about the coefficients of this representation (e.g. Milani, 2007). Instead, we assume that agents employ a misspecified forecasting rule; that is, in comparison to having all state variables as regressors, agents use only a subset of them or no states at all. We adapt the set-up in Malmendier and Nagel (2016) and specify agents' PLM as AR(1).[19] However, we assume that it does not include a constant so that agents know the true mean of the model. Consequently, the PLM of a generic variable $\mathbf{z}_t$ for a household in cohort k is given by:

$$\mathbf{z}_{t|k} = b^{\mathbf{z}}_{t-1,k}\mathbf{z}_{t-1} + \varepsilon^{\mathbf{z}}_{t|k} \ , \tag{18}$$

where $\varepsilon^{\mathbf{z}}_{t|k}$ is a disturbance term which is serially-uncorrelated with zero mean and constant variance, and $b^{\mathbf{z}}_{t-1|k}$ is the estimated parameter of household k at time $t-1$. In our model, agents form expectations on output and inflation. Hence, the set of variables on which agents form expectations is given by $Y^f \equiv \{y, \pi\}$ with $\mathbf{z}_k \in Y^f$.

A further crucial point is the amount of information individuals are able to incorporate when forming expectations. We simplify the learning process and assume individuals that form expectations at time t use only information available at time $t-1$. By doing so, we avoid a simultaneity problem that arises when agents use time t endogenous variables to forecast future realisations, which in turn affects the time t endogenous variables. Hence, the realisation of time t endogenous variables and the formation of expectations of time $t+1$ variables are not simultaneously determined.[20]

The last element of the PLM is how its coefficients develop over time. Let $\mathbf{I}_t$ be the information set on which households base their forecast at time t. As explained above the information set $\mathbf{I}_t$ includes all model variables up to $t-1$. Consequently, the formation of expectations occurs before the realisation of the endogenous variables included in Y^f_t such that $\tilde{E}^k_t(\mathbf{z}_t) = \tilde{E}^k(\mathbf{z}_t|\mathbf{I}_t) \neq \mathbf{z}_t$. Instead, using equation (18), and presuming that the law of iterated expectations holds for the subjective expectations, households in cohort k forecast:

$$\begin{aligned}\tilde{E}^k_t(\mathbf{z}_{t+1}) = \tilde{E}^k_t\left(b^{\mathbf{z}}_{t|k}\mathbf{z}_t\right) &= \tilde{E}^k_t\left(b^{\mathbf{z}}_{t-1|k}\mathbf{z}_t\right) \\ &= \tilde{E}^k_t\left(b^{\mathbf{z}}_{t-1|k}\left(b^{\mathbf{z}}_{t-1,k}\mathbf{z}_{t-1} + \varepsilon^{\mathbf{z}}_{t|k}\right)\right) = (b^{\mathbf{z}}_{t-1|k})^2\mathbf{z}_{t-1} \ ,\end{aligned} \tag{19}$$

where for the first equality we use the PLM (dated $t+1$) and for the second that point estimates of the PLM parameters only include information up to $t-1$ (see Evans et al., 2013). The third equality makes use of the fact that agents form expectations before

[19] Orphanides and Williams (2005) and Slobodyan and Wouters (2012a) use similar specifications of agents' forecasting rule. The choice of order one is further consistent with a model under RE where variables are also Markov-processes of order one.

[20] This approach is also consistent with the literature (e.g. Evans and Honkapohja, 2012, Ch. 10.2.)

the current realisation of $\mathbf{z}$ such that also today's realisation is forecasted using the PLM. Finally, the last equality uses the fact that the PLM parameter estimated with information up to time $t-1$ is uncorrelated with the error term at time t, i.e. $\tilde{E}_t^k\left(b^{\mathbf{z}}_{t-1|k}\varepsilon^{\mathbf{z}}_{t|k}\right) = 0$. After the realisation of time t shocks, agents update their PLM parameters from $b^{\mathbf{z}}_{t-1|k}$ to $b^{\mathbf{z}}_{t|k}$ using the following recursive least-squares (RLS, henceforth) algorithm:

$$b^{\mathbf{z}}_{t|k} = b^{\mathbf{z}}_{t-1|k} + \gamma_{t|k}\left(R^{\mathbf{z}}_{t|k}\right)^{-1}\mathbf{z}_{t-1}\hat{\varepsilon}^{\mathbf{z}}_{t|k} \tag{20a}$$

$$R^{\mathbf{z}}_{t|k} = R^{\mathbf{z}}_{t-1|k} + \gamma_{t|k}(\mathbf{z}_{t-1}\mathbf{z}'_{t-1} - R^{\mathbf{z}}_{t-1|k})\ , \tag{20b}$$

for each $\mathbf{z} \in Y^f$. Here, $\hat{\varepsilon}^{\mathbf{z}}_{t|k} \equiv \mathbf{z}_t - b^{\mathbf{z}'}_{t-1|k}\mathbf{z}_{t-1}$ denotes the forecast error of cohort k and $\gamma_{t|k}$ gives the (potentially) age-dependent Kalman gain of cohort k that assigns the relative importance of $\hat{\varepsilon}^{\mathbf{z}}_{t|k}$ with respect to the previous estimate, $b^{\mathbf{z}}_{t-1|k}$ and $R^{\mathbf{z}}_{t-1|k}$ (with $R^{\mathbf{z}}_{t-1|k}$ being the covariance matrix of estimates). The lower the noise in the explanatory variables, the stronger the update. Importantly, we assume agents start with an initial set of PLM parameters that is drawn from a truncated normal distribution around the PLM parameter of estimated from a simulation of the economy with RE. We summarise the timing assumption in figure 2.

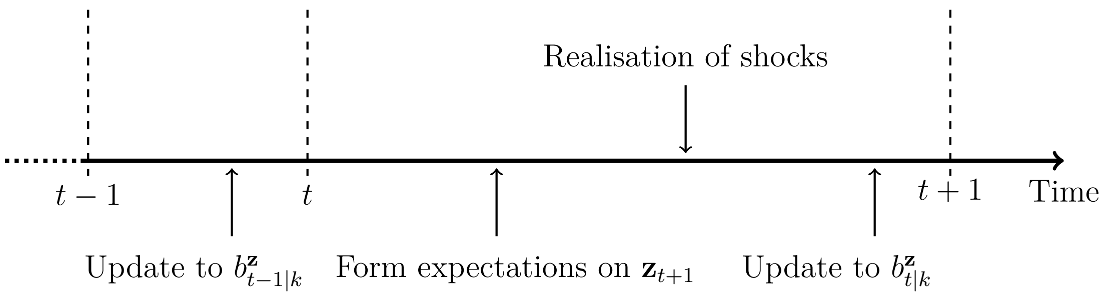

FIGURE 2: Timing Assumption of Updating the PLM Parameters

3.2. Experience-Based Learning

The novelty of EBL lies in the age-dependent form of parameter updating. Concerning this aspect we deviate from the CGL-literature that sets the gain $\gamma_{t|k}$ equal to a constant g. Under CGL all agents update equally such that there is no heterogeneity in their PLM parameters. However, Malmendier and Nagel (2016) provide evidence that the gain parameter $\gamma_{t|k}$ depends on the amount of lifetime data (or equivalently age), $t-k$, of individuals in cohort k.

$$\gamma_{t|k} = \begin{cases} \frac{\theta}{t-k} & \text{if } t-k \geq \theta \\ 1 & \text{if } t-k < \theta\ , \end{cases} \tag{21}$$

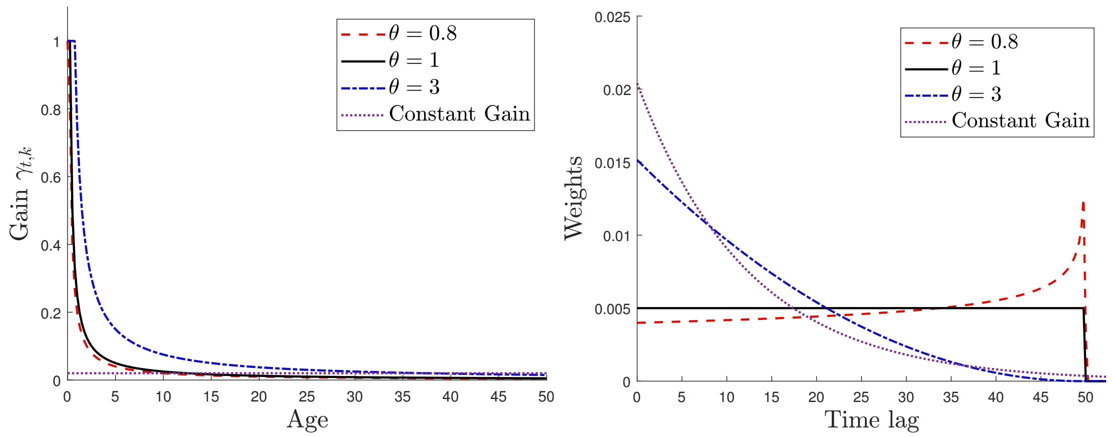

FIGURE 3: Gain and Weights on Past Data

Notes: The left panel denotes the evolution of the gain parameter over age (in years) for three different values of θ (the estimate of Malmendier and Nagel (2016) is around 3). The right panel shows how a 50 year old agents weights past information when estimating the parameters of the PLM (again for different θ). In both graphs the purple line denotes the case of CGL.

where $\theta > 0$ determines the degree to which individuals react to recent observations. Above specification implies, firstly, that expectations are heterogeneous between cohorts. Secondly, it implies that young agents have higher gains than older ones so that the updating of their PLM's parameters is more strongly compared to their older peers.
Both aspects are captured in figure 3. The left panel plots the gain parameter over age for different values of θ.[21] Young agents have high gains, consistent with the idea that they have less lifetime observations and, therefore, rely more on current data. The size of gains also decreases in age; the more so, the higher θ, which reflects that less weight is given to the more distant past. This is also captured in the right panel of figure 3. It shows the implied weights a 50 year (200 quarter) old individual puts on data observed over its lifetime for different values of θ.[22] For $\theta > 1$, data observed early in life receives negligible weights as an individual ages so that recent data is more important to update the PLM (data before birth has weight zero - only lifetime information is used). Both panels taken together imply that, at time t, agents of different age estimate the PLM's parameters differently, which results in heterogeneity in expectations across cohorts. Note also that agents of different age use a different amount of information.[23] Although in our perpetual youth structure there may still be individuals who use information from the far past, their fraction declines as time passes by. Further, the weight such an individual

[21]The graph is based on Malmendier and Nagel (2016). Appendix B shows how to derive figure 3.
[22]Note that RLS is the recursive formulation of weighted least squares (WLS, henceforth). The weights inside the weighting matrix contain the gain parameter $\gamma_{t|k}$ and, thus, depend on θ.
[23]Strictly speaking also newly entering agents would have access to all observations. However, as seen in the right panel of figure 3, individuals do not put any weight on observations before birth. This stands in contrast to the SL-literature in which individuals weigh *all* data points, no matter how old they are.

would put on this information would be very small, such that this informations' influence on the current aggregate expectation is negligible.

3.3. Constant-Gain Learning

A feature of EBL is that agents have different gain parameters depending on their age. As mentioned above, studies of DSGE models with dynamic non-rational expectations instead often employ learning algorithms with CGL so that all agents react equally to new observations. In practice this amounts to replacing $\gamma_{t|k}$ in (20) with a constant, g, such that the left panel of figure 3 shows a constant (purple line) across ages. Under this assumption, agents of different cohorts are homogeneous with respect to their expectation formation, i.e.

$$\tilde{E}_t^k \mathbf{z}_{t+1} = \tilde{E}_t \mathbf{z}_{t+1} = (b_{t-1}^{\mathbf{z}})^2 \mathbf{z}_{t-1}$$

for all cohorts k. This setup still retains the feature that new observations (and, hence, forecast errors) are weighted higher than old observations (see the right panel of figure 3). However, each cohort weights them equally. In the following we will interpret the CGL approach as the counterpart of EBL where we *shut off* the effect of experiences on individuals' expectations. We simulate our model for this specification in order to study the *additional* endogenous source of variation that stems from experience effects alone.

4. Quantitative Analysis

This section presents a quantitative analysis of the model with EBL. We start with a brief description of our parameter choices. Next, we simulate the model under EBL and discuss its basic properties. We compare these to a model in which agents update their parameter estimates with constant gain, which yields a key result we use for the subsequent analysis. Further, we compare the monetary policy stabilisation trade-off in a model with EBL to models with RE and CGL and explore how different relative sizes of young to old cohorts (we call this "demography", henceforth) affect results. Finally, the transmission of monetary policy shocks under EBL is investigated by computing impulse responses, which are compared to those from models with RE and CGL.

4.1. Parametrisation and Solution

Parameterisation. One period in the model corresponds to one quarter. We calibrate the model's deep parameters to U.S. data (Table 1 gives a summary). Our choice of the survival probability $\omega = 0.995$ is guided to meet an average life span of 200 quarters, which

Table 1: Parameter Choices (quarterly)

Variable		Value	
β	Discount factor	0.995	Galí (2015)
ε	Elasticity of substitution	9	Galí (2015)
α	DRS parameter	0.66	Galí (2015)
ξ	(Inverse) Frisch elasticity	2	own
ϕ	Rotemberg parameter	91.9	Galí (2015)
φ_π	Taylor parameter π	1.5	Galí (2015)
φ_y	Taylor parameter y	0.125	Galí (2015)
ω	Survival probability	0.995	own
ρ_x	Persistence $\hat{\epsilon}^x$	0.95	Smets and Wouters (2007)
ρ_m	Persistence $\hat{\epsilon}^m$	0.50	Galí (2015)
σ_x	Standard deviation ν^x	0.45	Smets and Wouters (2007)
θ	EBL parameter	3.044	Malmendier and Nagel (2016)
g	Gain under CGL	0.02	Milani (2007)
		Steady States	
π	Inflation	1	
x^*	productivity	1	

represents the working-life of an agent.[24] Most of the other parameters are taken from the textbook model of Galí (2015). The households' discount factor β is calibrated to get a steady state real *annualised* return on riskless bonds of about 4% given our choice for ω. Furthermore, we set the steady state elasticity of substitution to $\varepsilon = 9$, which implies a steady state mark-up of 12.5%. The parameter of the production function α is chosen to be 0.66 in line with the labor share in U.S. data. The Rotemberg adjustment cost parameter is chosen to match a fraction of non-adjusters of 0.75 in a model with Calvo price setting.[25] As is standard in the literature, we set $\varphi_\pi = 1.5$ and $\varphi_y = 0.125$. Further, we choose the serial correlation coefficient of the TFP shock as $\rho_x = 0.95$ and of the monetary policy shock as $\rho_m = 0.5$ as well as the standard deviation of ν_x as $\sigma_x = 0.45$ (for the MP IRFs we use other values as denoted below). We choose the learning parameter that governs the age-dependent gain under EBL as $\theta = 3.044$ (Malmendier and Nagel, 2016) and the CGL parameter g as 0.02 according to Milani (2007) and much of the learning literature.[26] Finally, steady state productivity and inflation are targeted to be one.

Solution. Each of these models is simulated for the same random sequence of TFP shocks, while setting the monetary policy innovation to zero. To initialise the PLM parameters for the learning models, we simulate the economy under REs for 120 quarters and estimate an AR(1) model for inflation and output. The estimated AR(1) coefficients

[24] Agents in our model are workers. We do not model retirement.

[25] If the fraction of non-adjusters is 0.75 (= average price duration of 4 quarters), $\phi = \frac{(\bar{\varepsilon}-1)0.75}{(1-0.75)(1-0.75\beta\omega)}$.

[26] See Figure 3 for how this choice affects EBL.

for both variables serve as the respective initial PLM parameter for the models with EBL and CGL. To start model simulations, we endow each cohort by the same initial persistence parameter, $b^z_{-1|k} = b^z_{-1}$, and the same covariance matrix of estimates $R^z_{-1|k} = R^z_{-1}$ for $z \in Y^f$ that are updated subsequently. We then simulate the economy for $T = T^b + 10,000$ quarters, where $T^b = 300$ is the number of periods that are discarded in order to wash out the impact of the initial values from the simulation of the RE economy. Each period, a member of cohort k updates its parameter estimate and the covariance matrix according to equation (20). Similar to Slobodyan and Wouters (2012a), we invoke a projection facility and restrict the new estimate to induce a stationary AR(1) process which requires $|\delta^z_{t|k}| < 1$ for all cohorts k and $z \in Y^f$.[27] Evans and Honkapohja (2012) argue that agents want to avoid explosive paths of the economy such that the agent chooses its parameter estimate accordingly. Each new born cohort draws its initial parameters from a truncated normal distribution around the RE estimate, where the normal distribution is truncated at ± 1. Further, since there is a infinite number of cohorts in this economy, we need to choose a finite number of cohorts for our simulation exercise. A high number of cohorts reduces the approximation error but is subject to the curse of dimensionality. Since the baseline model calibrates the survival probability such that the expected lifetime is 200 quarters, we restrict the number of cohorts in the aggregation to be 200 and normalise cohorts weights to sum to one. A detailed description of the algorithm is found in Appendix C.

4.2. Dynamic Properties of EBL

In this section we point out the properties of EBL that will be important for the interpretation of the main results. In particular, we show that under EBL, the perceived persistence in inflation and output are *on average* lower compared to an economy where we shut-off experience effects. Further, we show that the model provides a meaningful amount of expectation heterogeneity in general equilibrium.

As pointed out in Malmendier and Nagel (2016), an important feature of EBL is that young agents' beliefs are more sensitive towards new observations compared to those of older individuals. In our model, the source of heterogeneity in expectations across cohorts stems from the heterogeneity in individuals' perceived persistence in both inflation and output. To see this, consider the expectation difference between a young ($age = 10$ quarters) and an old ($age = 158$ quarters) individual:

$$\tilde{E}^{k=10}_t (\pi_{t+1}) - \tilde{E}^{k=158}_t (\pi_{t+1}) = \left[\left(b^{\pi}_{t-1,k=10} \right)^2 - \left(b^{\pi}_{t-1,k=158} \right)^2 \right] \pi_{t-1} \ , \tag{22}$$

[27] The restriction is invoked in only 6% of updates for PLM parameters under EBL. This falls to 2% when we marginally increase the bound on parameter estimates. Its usage is then similar to the one in Slobodyan and Wouters (2012a).

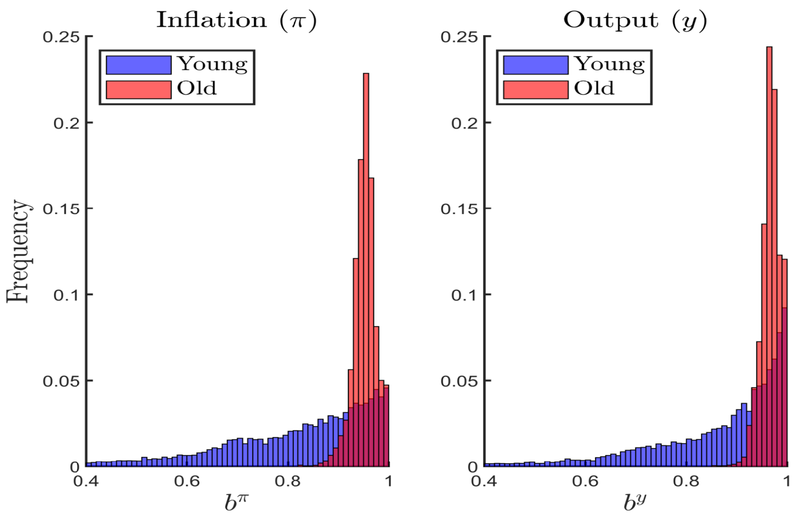

FIGURE 4: Distribution of PLM Parameters in the Model with EBL

Notes: The blue histogram denotes the distribution of PLM parameters for young ($k = 10$) individuals while the red histogram shows the distribution of the PLM parameters for old ($k = 158$). The left part shows the PLM parameter for inflation while the right part shows the PLM parameter for output. We simulate the economy for 10,000 periods.

where we use the cohort forecast as in (19) in which both agents use aggregate inflation. However, agents of age group k differ by the perceived persistence of inflation estimated from data up to period $t-1$: $b^{\pi}_{t-1,k}$. In Figure 4 we plot the ergodic distribution of PLM parameters in the EBL model for individuals of cohort $k = 10$ and $k = 158$ for $10,000$ simulated periods. The left part of Figure 4 shows the distribution of PLM parameters for inflation, while the right part shows the distribution of PLM parameters for output. Note that the distribution of the PLM parameters is bounded above by one due to the projection facility invoked as described in the previous section. Figure 4 demonstrates that the perceived persistence in both output and inflation of young agents are *on average* more dispersed and smaller relative to the one of old agents. Recall that young individuals rely on a lower amount of information when updating their PLM parameters and are more sensitive towards new observations. On the one hand, the variance in individuals' parameter estimates decreases in the number of observations used for updating. On the other hand, since young individuals are relatively more sensitive towards any new observation when updating their PLM parameters, the latter are more dispersed compared to the ones of old individuals.

Equation (22) states that the heterogeneity in individuals' perceived persistence then directly translates into heterogeneity in expectations across cohorts. In Figure 5 we plot the ergodic distribution of the difference in equation (22) over $10,000$ simulated periods. The EBL model generates a considerable amount of heterogeneity in expectations ranging up to 1.5% points deviation from steady state. Moreover, the mean value of this difference is slightly negative, which indicates that, *on average*, individuals of different cohorts dis-

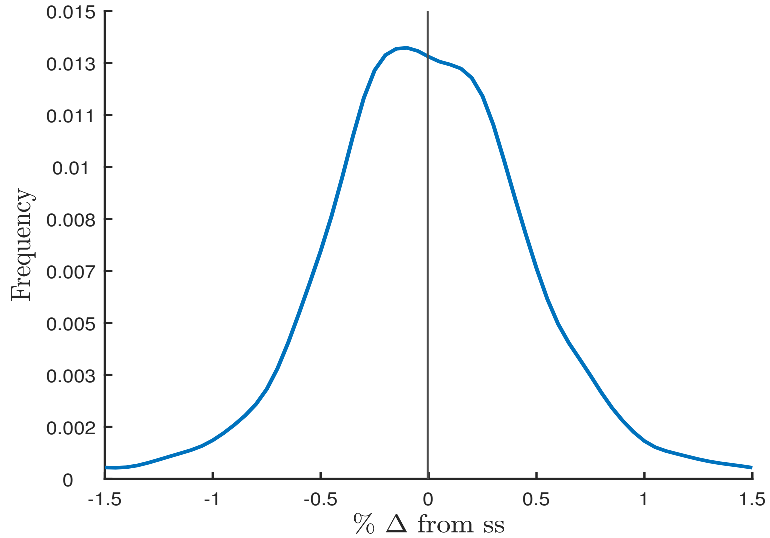

FIGURE 5: Cohort Inflation Expectation Differences

Notes: The blue line denotes the distribution of differences in expectations between young and old ($\tilde{E}^Y\pi - \tilde{E}^O\pi$) over time under the baseline calibration in Table 1. We focus on the 1^{th} to 99^{th} percentiles.

agree on inflation expectations.

The dynamics of this difference in expectations is linked to the experienced inflation of each cohort. Figure 6 displays a snapshot of one simulation path. The blue thick line depicts simulated realised inflation (left y-axis) and the red lines denote the slope of the PLM for a young (dashed) and an old (dotted) cohort (right y-axis). Two aspects stand out. First, movements in *actual* inflation are reflected in PLM parameters with a lag which is a consequence of the backward-looking PLM-specification and lagged updating. Recall that the update step only uses information up to the last period, so that any shift in inflation is included with a one-period lag. Second, PLM parameters of young and old households differ, which, in turn, determines differences in expected inflation (figure 5). The reason for this result is as follows: consider the region around $t = 80$. During the last 10 periods inflation fell constantly and at $t = 80$ nearly reaches the zero-steady state, which gives the young cohort a very different information set compared to the old one that also experienced the prolonged period of above steady state inflation between $t = 50-70$. The young, therefore, expect inflation to be smaller as indicated by a reduced PLM parameter. In contrast, old agents view inflation to be highly persistent (the dotted line remains at a high level compared to the dashed line).

The heterogeneity in the perceived persistence in output and inflation expectations affects the *aggregate* perceived persistence in the economy. As for aggregate expectations, the *aggregate* perceived persistence is a weighted-average over cohort values. In Figure

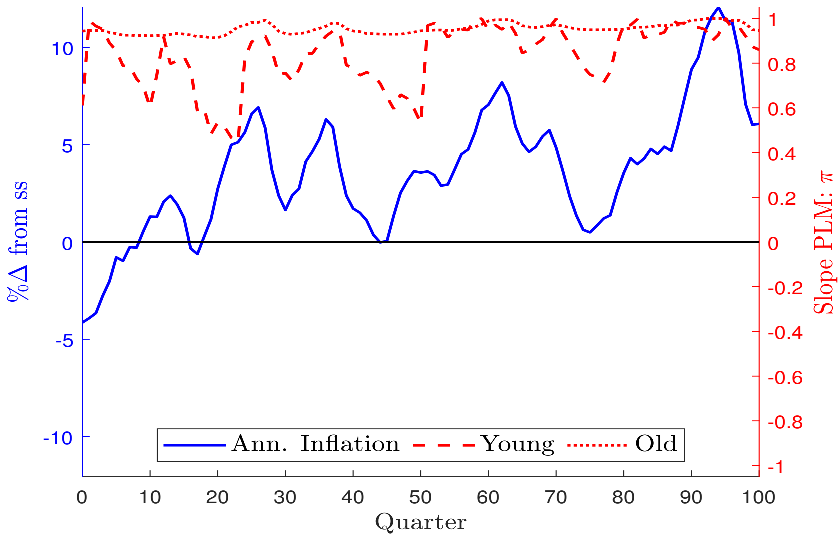

FIGURE 6: Actual Inflation and PLM Parameters of Young and Old

Notes: The blue line depicts the annualised realised inflation deviation from steady state under EBL. The red lines show the estimated slope of the PLM for young (dashed) and old (dotted) cohorts. Throuhgout the simualtion we only compare agents of ages 10 and 158. Hence, we do not follow two single age groups through time.

7 we compare the distribution of the aggregate perceived persistence under EBL to the one obtained when we shut off the experience effect and assume that individuals in each cohort attach the same weight to new observations. In the latter model, the perceived persistence in expectations is *on average* higher and less dispersed when experiences do not affect the way individuals update their PLM parameters. Hence, in the EBL model (age-related) heterogeneity in expectations pushes down the aggregate perceived persistence in inflation and output.[28] As we show in the next section, this affects the *impact* response of inflation and output to shocks, which in turn affects the pass-through of monetary policy on aggregate demand.

To sum up: in our model the heterogeneity in expectations stems from the heterogeneity in individuals' perceived persistence in inflation and output. Since young individuals use less information when updating their PLM parameters and are more sensitive towards any new observation, their perceived persistence is lower *on average*, which pushes down the aggregate perceived persistence in the economy. Shutting off the experience effect on individuals' updating of PLM parameters increases the aggregate perceived persistence in the economy and decreases the dispersion in the PLM parameter estimates. From this observation, we now turn to the study of monetary policy.

[28]Our results depend on the choice of θ. Malmendier and Nagel (2016) provide different estimates. We use the result of their preferred specification, which is also the lowest estimate for θ. In Appendix D, we check the robustness of our results against setting θ to the highest value provided by the authors. Our findings intensify so that our results are rather a lower bound.

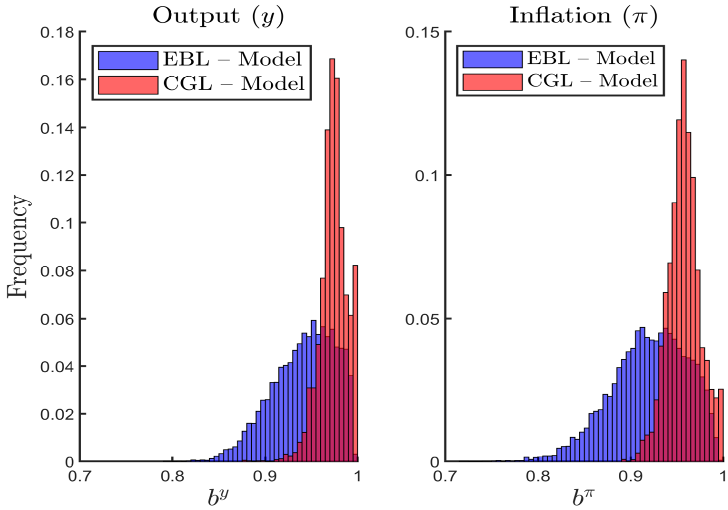

FIGURE 7: Distribution of PLM Parameters in the Models with EBL and CGL

Notes: The blue histogram denotes the distribution of PLM parameters in the model under EBL while the red histogram shows the distribution of the PLM parameters under CGL. The left part shows the PLM parameter for output while the right part shows the PLM parameter for inflation. We simulate the economy with EBL and CGL for 10,000 periods.

4.3. The Effect of EBL on Monetary Policy

In this section we study the effect of EBL on monetary policy and relate the magnitude of this effect to the age-distribution. In particular, we consider how EBL affects the trade-off monetary policy faces when aiming to stabilise inflation and output under supply shocks. Further, we analyse the effect of EBL on the transmission of monetary policy shocks.

4.3.1. Trade–off under Supply Shocks

First, we compare different Taylor rule calibrations with respect to their ability to stabilise output and inflation under TFP shocks. In the context of the NK model, it is a well-known result that the Taylor rule is incapable to stabilise output and inflation at the same time when TFP shocks perturb the economy (e.g. Galí, 2015). Starting from the baseline parametrisation of the Taylor rule parameters (φ_π, φ_y), we successively increase monetary policy's output stabilisation motive (increase φ_y up to 1), while holding constant the Taylor parameter on inflation ($\varphi_\pi = 1.5$). The results are shown in Figure 8 where we plot the standard deviation of output (σ_y) against the standard deviation of inflation (σ_π) for each combination of (φ_π, φ_y). The blue line displays the simulation results under EBL, the black line shows corresponding results when we shut off the experience effects (CGL–Model) and the red line displays the simulation results under RE.
There are two main results. First, the standard monetary policy trade-off of stabilising output and inflation under supply shocks prevails under EBL. As we increase the Taylor coefficient on output, the volatility of output decreases, while the volatility in inflation

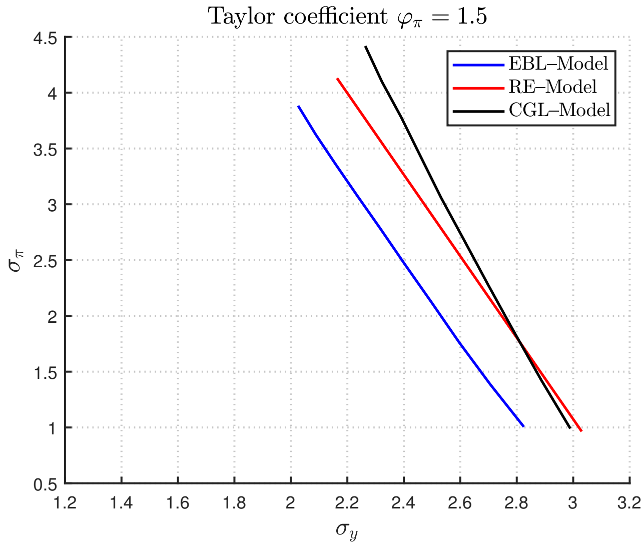

FIGURE 8: Monetary Policy Frontier under a TFP–shock

Notes: We define a grid of points for φ_y. We then simulate the economy as described above for $10,000$ periods for each grid point while holding $\varphi_\pi = 1.5$. We then plot the standard deviation of output against the one for inflation for the models under EBL (blue), CGL (black) and with RE (red).

increases.

Second, the policy frontier shifts inwards under EBL which implies that for each combination of the Taylor coefficients, output is *less* volatile under EBL. There are two effects that explain this result. The first one is that the pass-through of monetary policy on aggregate demand under the supply shock is weaker under adaptive expectations *in general*. Under RE, agents are forward-looking. Hence, they internalise the Taylor rule's impact on the future path of real interest rates when forming expectations. This directly affects current output via the Dynamic IS Curve.

Under EBL (and CGL), in contrast, agents are backward-looking. They fail to project the effect of monetary policy on the future path of the real interest rate. Instead, movements in the nominal interest rate affect expectations only with a lag. Intuitively, movements in the nominal interest rate affect current inflation and output, which, in turn, are used to form expectations on inflation and output only *in the next period*. As a consequence, the effect of the real interest rate on aggregate demand is subdued which, in turn, dampens the effect of aggregate demand on inflation (relative to the model with RE). Put differently, the reaction of the *real* interest rate will not necessarily affect output in the direction needed to stabilise inflation. Hence, the lower pass-through of monetary policy on aggregate demand lowers the volatility in output. However, for an increasing focus of the policy-maker on output stabilisation, we observe the line denoting CGL to be above the one for RE. Hence, in these cases output is *more* volatile despite adaptive expectations. Again, under RE, agents internalise the simple interest rate rule when forming expectations and understand how the nominal interest rate varies with fluctuations in inflation and output.

Thus, any variation in the weight on output moves expectations of the real interest rate one for one. Instead, under adaptive expectations, monetary policy affects individuals' expectations only indirectly and with one period delay. It is this weakened ability to influence agents' expectations that reduces the *relative* reduction in output volatility under adaptive expectations, which is the reason why the slope of the policy frontier under CGL is steeper. However, since the policy frontier under EBL is *always* below the one under RE, there needs to be another effect that is missing under age-independent CGL.

This second effect stems from the fact that, *on average*, the model with EBL displays lower perceived persistence in both inflation and output relative to the model where we shut off the experience effect (model with CGL). As discussed in the previous section, the inclusion of experiences attenuates the impact response of both inflation and output precisely because the perceived persistence under EBL is sufficiently reduced relative to the model with CGL. Hence under EBL, the policy frontier shifts downwards because the lower perceived persistence additionally mutes both the impact response of macroeconomic variables to TFP shocks and the lagged feedback effect of expectations on current variables.[29]

Demography. Next, we consider how the policy frontier is affected by a change in the demographic structure of the economy. In the perpetual youth model, a change in the demographic structure corresponds to a change in the survival probability, ω. To illustrate the effect of the demographic structure we substantially reduce the survival probability to 0.9. A change in the survival probability affects,

1. the effective discount factor, $\tilde{\beta} \equiv \beta\omega$
2. the slope of the Phillips-Curve (via the change in $\tilde{\beta}$)
3. *under EBL*, aggregate expectations, $\bar{E}_t$.

For the sake of the argument, we shut-off channels 1. and 2.. We do so by varying β such that $\tilde{\beta}$ stays constant and only consider the variation that stems from channel 3.. Recall that an increase the share of young agents further reduces the aggregate perceived persistence in the EBL-Model. As can be seen in Figure 9, this reduction in the aggregate PLM parameter further shifts the policy frontier downwards. We call this demography-driven shift of the policy frontier that stems from channel 3. the *Experience Channel*. Moreover, the slope of the policy frontier flattens which indicates that a higher weight on the Taylor coefficient on output increases inflation volatility by less compared to model

[29] Appendix E.2 performs the same analysis for a higher focus on inflation. Our results remain unchanged. Further, Appendix E.4 compares EBL to CGL with different gain parameters. While one can find a gain parameter under CGL to have a similar policy frontier, such a value is empirically implausible. Moreover, effects of demography as described below do not exist.

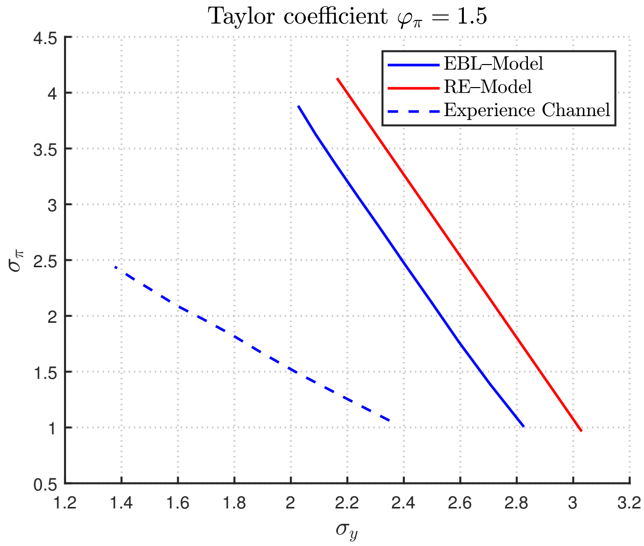

FIGURE 9: Monetary Policy Frontier under a TFP–shock: the Experience Channel

Notes: In addition to the remarks on Figure 8: we increase the share of young agents while simultaneuosly holding the effective discount rate $\tilde{\beta} = \beta\omega$ constant. Any changes to the baseline case of EBL are then attributable to differences in expectation formation in a young economy.

under baseline calibration. In other words, the trade-off becomes weaker because for the same reduction in output volatility, the increase in inflation volatility is lower. Hence, in an economy with a high share of young individuals, the monetary policy trade-off between stabilising output and inflation weakens. Intuitively, in an economy with a higher share of young individuals, the perceived persistence in inflation is lower such that the effect of inflation expectations on current inflation is muted.[30] This reduces both the impact response of inflation and the lagged effect of shocks through inflation expectations on current inflation.

The contribution of the Experience Channel to the total effect of a change in ω is substantial. In Figure 10, the red line denotes the simulation results if we consider the total effect of a decrease in ω to 0.9 by turning on channel 1. and 2.. The total effect is even a bit more pronounced as the effective discount factor is decreasing, which attenuates the effect of inflation expectations on current inflation even further. However, the total effect is mainly driven by the Experience Channel. In Appendix E.1, we show that if we shut off the experience channel, the demography-driven shift in the policy frontier is substantially less pronounced and stays close to the policy frontier obtained under RE.

[30] Recall that the trade-off between inflation and output volatility when increasing the MP-focus on output results from an interest rate movement that is directed less in a way that would counter the initial inflation response. RE would internalise the impact on future inflation which increases contemporaneous inflation. Yet, if expectations are backward-looking this channel is shut-off. If in addition, any change in current inflation induces a weaker inflation response, we observe an even weaker change in current inflation.

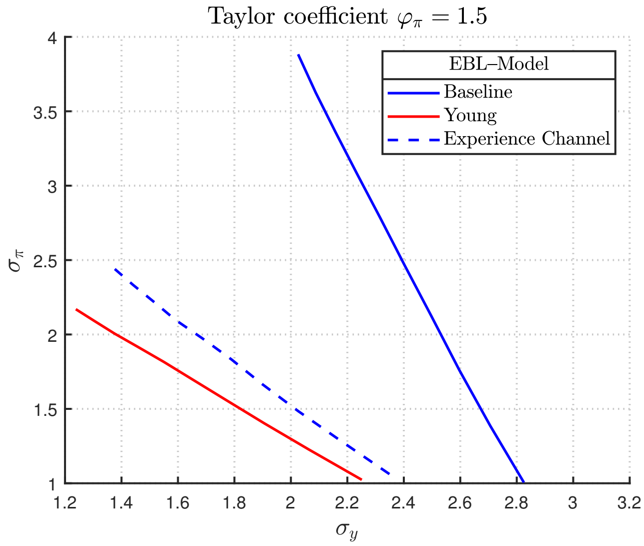

FIGURE 10: Monetary Policy Frontier under a TFP–shock: Decomposition under EBL

Notes: In addition to the remarks on Figure 9: we show the frontiers for the models under EBL in an old economy (blue solid) and in a young economy. The latter is divided into the full effect (red) and into the effect from experiences alone (blue dashed).

Hence, the Experience Channel is an important component to keep in mind when talking about the trade-off between output and inflation stabilisation. If the share of old individuals increases, the policy frontier shifts upward indicating that the pass-through of monetary policy on aggregate demand increases. However, the trade-off monetary policy faces when stabilising both output and inflation under supply shocks gets more severe as the share of old individuals increases. For the conduct of monetary policy this implies an inflation-fighting focus when the economy ages. In these economies, agents have a high perceived persistence of inflation and react little to new observations. Hence, not only becomes influencing inflation expectations harder but also the stabilisation trade-off is more severe.

4.3.2. Transmission of Monetary Policy Shock

Next, we analyse the effect of EBL on the transmission of monetary policy (MP) shocks and how this effect depends on the age distribution. To explore this we compute *generalized* impulse response function. We again simulate the economy under rational expectations to compute the initial parameter for the model under learning. We then simulate the economy under a TFP shock, while setting the monetary policy shock to zero. Next, we simulate the economy under the *same* path of TFP shocks and introduce an innovation to the monetary shock at time T^b. We then take the difference between these two series as the impulse response function to a monetary policy shock.

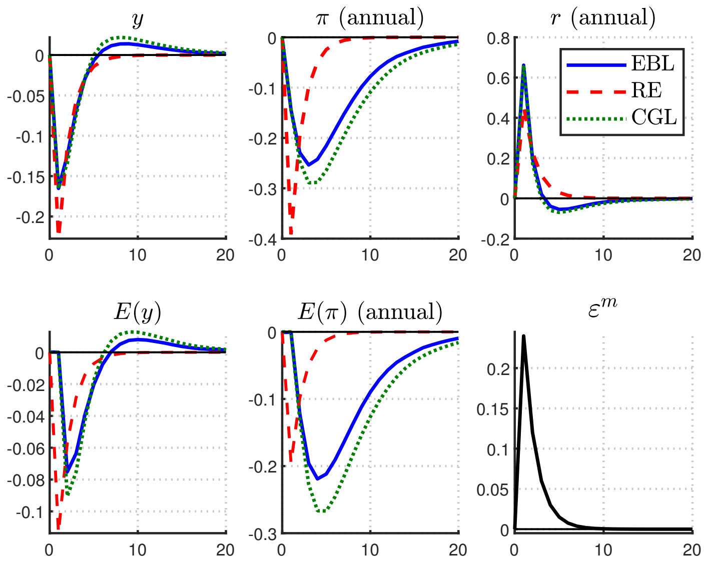

FIGURE 11: IRFs to a MP Shock

Notes: We show the IRFs for key variables in the economies under RE (red), CGL (black) and EBL (blue). We average responses over $8,000$ iterations to generate mean impulse responses. Output, output expectations as well as the monetary policy shock are measured as percentage deviations from their respective steady state while the other variables are measured as (annualised) deviation from their respective steady state.

Impact. Consider Figure 11. On impact, a contractionary MP shock affects both output and inflation negatively. In response, the central bank pushes the nominal interest rate down. This decrease is, however, not sufficient to offset the exogenous shift such that the real interest rate increases, on impact. Comparing the initial responses, both output and inflation react less under EBL compared to the RE model which again is the result of the muted pass-through of monetary policy on aggregate demand and – through the New Keynesian Phillips Curve – on inflation. If we shut off the experience effects on expectations (CGL–Model), we observe a similar response on impact of both, output and inflation, that becomes stronger the period after the shock hits the economy as expectations adjust.

Revision of beliefs. Under both, EBL and CGL, individuals revise their beliefs with a delay of one period. This backward-looking behaviour is visualised in the lower half of Figure 11, which illustrates responses of expectations. In the period *after* the shock, individuals revise beliefs on future inflation and output downwards, which feeds back into current output and inflation and creates hump-shaped responses in Figure 11. The lower impact response of inflation and, to a smaller extent, output under EBL is again explained by the lower perceived persistence of these two variables (see Figure 7). For output the simultaneous decrease in the real rate (which ceteris paribus increases demand) partly countervails the impact of more negative CGL-expectations. RE on output react stronger than those based on CGL or EBL. Intuitively, the persistence of the shock increases the nominal rate also for the periods to come, which implies smaller demand so that expected output is lower. As a consequence, also expected inflation is reduced compare to the economies with adaptive expectations in which agents are incapable of perceiving these changes due to their backward-lookingness.

Dynamics. The dynamic response of macroeconomic variables under EBL is quite different compared to the one under RE. While under RE the economy reverts back to the steady state roughly after ten quarters, deviations in the economy under EBL are much more persistent. This slower reversion to the steady state results from the lower pass-through of monetary policy on aggregate demand. Since monetary policy is less effective in stabilising aggregate demand under adaptive expectations, the effect of the MP shock is prolonged compared to the economy under RE. Moreover, under EBL, agents revise their beliefs downwards and the shock is more slowly transmitted into their expectations compared to RE where individuals know the actual data generating process and perfectly internalise the shock process into their expectations. These two effects taken together explain the higher persistence in the response of inflation and output under EBL relative to the economy under RE. In addition, inflation displays a hump-shaped response under EBL which results from the backward-looking behaviour of agents and the lagged response of expectations to the monetary policy shock. In turn, output does not display this hump-shaped behaviour but starts reversing to its steady state value directly after the initial response. Hence, notwithstanding the reduced pass-through of monetary policy on aggregate demand, the decrease in the real interest rate is sufficient to reverse the drop in output in the first period after after the MP shock hits the economy.
To bring inflation back to its steady state value, monetary policy is decreasing the nominal interest rate by that much that the real interest rate undershoots, which, in turn, results in an overshooting of output which further boosts inflation upwards towards its steady state level. This overshooting is more pronounced if we shut off the experience effect which explains the closing gap in the inflation response of the models under EBL and CGL.

Demography. Next, we again lower the survival probability ω to 0.9 and analyse the effect of the resulting change in the age-distribution on the transmission of the monetary policy shock.[31] In Figure 12 we consider the effect of a change in the age-distribution that results from the Experience Channel 3. alone. The solid blue line denotes responses in the baseline (old) economy and the blue dotted line shows the impulse response function under EBL when the survival probability is lowered to 0.9 and only affects the economy

[31] In Appendix E.3 we perform the same exercise for the models under RE and CGL. Given no experience channel exists, differences between a young and old economy are small.

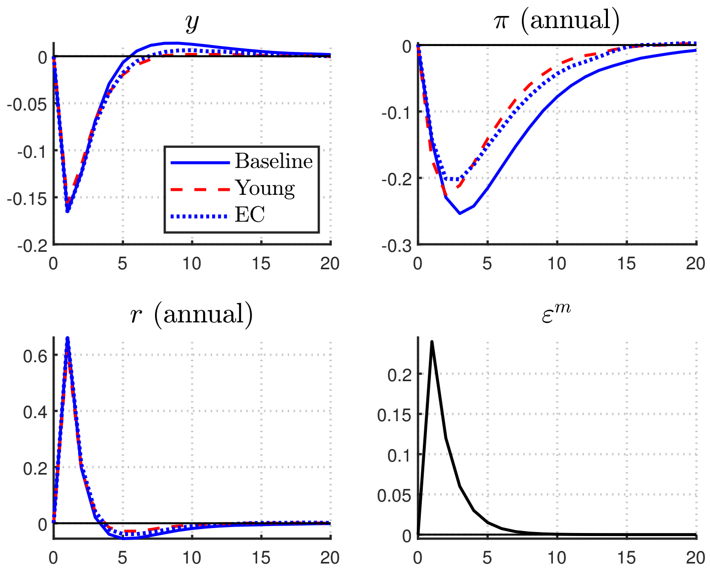

FIGURE 12: IRFs to a MP Shock: Decomposition under EBL

Notes: We show the IRFs for key variables in the economies under RE (red), EBL (blue solid) and when isolating the experience channel (blue dotted). We average responses over $8,000$ iterations to generate mean impulse responses. Output and the monetary policy shock are measured as percentage deviations from their respective steady state while the other variables are measured as (annualised) deviation from their respective steady state.

through the Experience Channel (channel 3.). In contrast the dashed red line denotes total responses when channels 1. to 3. are active. Recall that channel 1. reduces the impact of expected on current inflation and channel 2. increases the slope of the NKPC. Given the output reduction, the latter should amplify the shock compared to the baseline scenario.

As discussed in Section 4.2, an increase in the share of young agents (i.e. decrease in ω) reduces the aggregate perceived persistence of both inflation and output in the economy. This is the only channel active on the blue dotted line. The Experience Channel accounts for most of the total response (red), since the decrease in the perceived persistence reduces the impact response of both inflation and output to the contractionary MP shock. This muted response is more pronounced for annualised inflation, which again stems from the fact that the pass-through of monetary policy on aggregate demand is weakened, which limits the impact of aggregate demand on prices (via the NKPC). Due to the sluggish response in inflation, monetary policy pushes down the nominal interest rate sufficiently such that output overshoots. However, the overshooting is less pronounced which is due to the fact that the perceived persistence is lower in the economy with a higher share of young individuals. After period four, the Experience Effect is markedly compensated by channels 1. or 2., as the red line lies above the dotted blue line. Channel 2. implies an increase in the slope of the NKPC so that the quick recovery of output leads to a quicker return to the steady state for inflation. However, the Experience Channel still accounts for most of the response.

To sum up: the response of inflation and output to a MP shock is weaker but more persistent under EBL relative to a model with RE. Taking into account experience effects reveals an additional source of variation through the age-related heterogeneity in expectations that depends on the age-distribution in the economy. In the EBL model, a change in the age-distribution affects the aggregate perceived persistence in both output and inflation and thereby the aggregate expectations. This Experience Channel is absent under RE. If the share of young agents increases, the Experience Channel further mutes the impact response to a MP shock since the higher share of young individuals pushes down the aggregate perceived persistence. We further find that the Experience Channel substantially contributes to the effect of a change in the age-distribution on the transmission of monetary policy shocks.

5. Conclusion

This paper aims to answer the question how *age*-dependent heterogeneity in expectations affects monetary policy's stabilisation trade-off under supply shocks and the transmission of monetary policy shocks. To address this issue, we introduce a NK model with overlapping generations in which individuals use a simplified model to forecast inflation and output. Motivated by the empirical evidence of Malmendier and Nagel (2016), individuals' expectations depend on their respective life-time experiences, which, due to the presence of differently aged cohorts, creates heterogeneity in expectations.

The source of expectations heterogeneity in our model is the heterogeneity in the perceived persistence across cohorts. We find that under EBL, the *aggregate* perceived persistence in the economy is pushed down relative to a model where we shut off experience effects and assume that agents have the same perceived persistence. This downward shift in the aggregate perceived persistence under EBL reduces the impact response via expectations to exogenous shocks of both inflation and output relative to a model without experience effects.

We find that the downward shift in the aggregate perceived persistence under EBL weakens the stabilising power of simple monetary policy rules relative to a model with RE. This mainly stems from a weaker pass-through on aggregate demand which is more pronounced under EBL relative to a model without experience effects. In comparison to the model with RE, output is always less volatile under EBL. Moreover, we find that the trade-off between stabilising output and inflation under supply shocks turns stronger as the economy ages. While this finding is also present under RE, it is less pronounced compared to the model with EBL. In the latter model, the increase in the share of old agents pushes the aggregate perceived persistence upwards which increases both the impact response of output and inflation to exogenous shocks and the (lagged) response of expectations to past monetary policy actions. This Experience Channel is absent under RE. Furthermore, comparing the impulse responses to a monetary policy shock, we observe the demographic structure to have negligible effect on the shock transmission using RE, but to imply substantial differences in the response of inflation responses under EBL. In the latter model, inflation responds stronger and more persistently in an old economy, in which the aggregate perceived persistence of inflation is, on average, higher than in a young economy.

While we focused on understanding the channels by which EBL affects monetary policy that uses simple policy rules to stabilise fluctuations of economic variables, there are several worthwhile extensions. For example, one may estimate the model with Bayesian methods to identify the parameter that Malmendier and Nagel (2016) estimated in their econometric analysis. Estimating the shape parameter for cohort gains θ from a DSGE model would clarify the strength of EBL. One could further scrutinise whether and how differences in inflation expectations contribute to heterogeneous consumption responses across generations after a MP shock. Recently, Berg et al. (2020) find that the demographic structure matters for the transmission of MP shocks due to their heterogeneous effects on the consumption responses across age groups. Another relevant question is to further understand the effect of experience-based heterogeneity on the conduct of monetary policy. Based on an ad-hoc loss function one could compare which Taylor rule parametrisations are most welfare-improving. However, while the comparison of (ad-hoc) specified Taylor rules is insightful, these are not derived from preferences, do not account for private agents form of expectation formation, and, hence, do not represent optimal policy (see Gasteiger, 2014; Di Bartolomeo et al., 2016).

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Appendix

A. Model Derivations

A.1. Optimisation Problem of the Household

Households optimise discounted life-time utility:

$$\tilde{E}_t^k \sum_{j=0}^{\infty} (\beta\omega)^j \left[\ln\left(c_{t+j|k}\right) - \psi_n \ln(1 - l_{t+j|k})\right] \ ,$$

subject to the budget (2) and the initial condition $b_{t+j-1|t+j} = 0$. The Lagrangian reads:

$$\begin{aligned}\mathcal{L} = \max_{\{c_{T|k}, l_{T|k}, b_{T|k}, \lambda_{T|k}\}_{T=t}^{\infty}} \ & \tilde{E}_t^k \sum_{T=t}^{\infty} (\beta\omega)^j \Big\{\ln\left(c_{T|k}\right) - \psi_n \ln(1 - l_{T|k}) \\ & + \lambda_{T|k}\left(\frac{r_{T-1}}{\omega} b_{T-1|k} + p_T w_T l_{T|k} + \mathcal{D}_T - p_T c_{T|k} - b_{T|k}\right)\Big\} \ ,\end{aligned}$$

where we already replaced $z_{t|k} = \frac{1-\omega}{\omega} b_{t-1|k}$. First-order conditions for cohort k are given by

$$0 = \frac{1}{c_{T|k}} - \lambda_{T|k} p_T \tag{23}$$

$$0 = -\lambda_{T|k} + \beta\omega \tilde{E}_t^k \left(\lambda_{T+1|k} \frac{r_T}{\omega}\right) \tag{24}$$

$$0 = -\psi_n \frac{1}{1 - l_{T|k}} + \lambda_{T|k} w_T p_T \ . \tag{25}$$

Inserting (23) into (24) and setting $T = t$:

$$1 = \tilde{E}_t^k \left\{Q_{t,t+1|k} r_t\right\} \ ,$$

where $Q_{t,t+1|k} \equiv \beta \frac{p_t c_{t|k}}{p_{t+1} c_{t+1|k}}$ denotes household's SDF.Using (23) and (25) gives the labour supply of household k:

$$\psi_n \frac{c_{t|k}}{1 - l_{t|k}} = w_t \ .$$

A.2. Derivation of firm decisions

A.2.1. Demand of the Retailer

In this appendix we derive the final good firm's demand for intermediate inputs and the price aggregator.
Production follows a Dixit-Stiglitz technology that uses all intermediate inputs:

$$y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \ ,$$

with ϵ_t being the elasticity of substitution within the basket of intermediate goods. Consider the profit maximisation problem of the final good firm:

$$\Pi_t^F = \max_{y_{i,t}} \quad p_t y_t - \int_0^1 p_{i,t} y_{i,t} di = p_t \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p_{i,t} y_{i,t} di \ .$$

Maximising the profit Π_t^F with respect to $y_{i,t}$ yields final good firm's demand for intermediate good i:

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t} \right)^{-\varepsilon} y_t \ .$$

To derive the price level we use the condition of zero-profits and above demand equation:

$$p_t = \left(\int_0^1 p_{i,t}^{1-\varepsilon} di. \right)^{\frac{1}{1-\varepsilon}} \ . \tag{A.1}$$

A.2.2. Optimisation Problem of the Intermediate Firm

Cost minimisation problem. First, consider the cost minimisation problem of intermediate good firm, i:

$$\mathcal{C}(l_{i,t}) = \min_{l_{i,t}} p_t w_t l_{i,t} \quad \text{subject to} \quad y_{i,t} \leq x_t l_{i,t}^{\alpha}$$

The Lagrangian reads:

$$\mathcal{L} = -\left(p_t w_t l_{i,t}\right) + \mu_t \left(x_t l_{i,t}^{\alpha} - y_{i,t} \right) \ .$$

The first order condition w.r.t. $l_{i,t}$ is given by:

$$p_t w_t = \mu_t \alpha x_t l_{i,t}^{\alpha-1}$$

where the Lagrange Multiplier μ_t can be interpreted as firm i's nominal marginal cost of a further unit of production input. Let, $\mathrm{mc}_t^r \equiv \frac{\mu_t}{P_t}$ be firm i's real marginal cost, then firm i's labour demand is given by

$$l_{i,t} = \left(\frac{w_t}{\alpha \mathrm{mc}_t^r x_t}\right)^{\frac{1}{\alpha-1}} \tag{A.2}$$

Since this is the same condition for all firms, it follows that $l_{i,t} = l_t$ for all i. Then, firms' real marginal costs are given by:

$$\mathrm{mc}_t^r = \frac{1}{\alpha}\frac{w_t l_t^{1-\alpha}}{x_t} \tag{A.3}$$

Optimisation under costly price adaption. Consider the maximisation problem of an intermediate good firm i. The firm maximises the sum of discounted real profits $d_{i,t}$ subject to the demand of the final good firm and its production function. The maximisation problem is given by:

$$\begin{aligned}
&\max_{p_{i,t+j}} \quad \bar{E}_t \sum_{T=t}^{\infty} Q_{t,T} d_{i,T} \\
&\text{s.t} \quad y_{i,t} \leq \left(\frac{p_{i,t}}{p_t}\right)^{1-\varepsilon} y_t \\
&\quad y_{i,t} \leq x_t l_{i,t}^{\alpha}
\end{aligned}$$

where $Q_{t,T}$ is the *real* aggregate stochastic discount factor of households and $d_{i,t}$ is given by:

$$d_{i,t} = \frac{p_{i,t}}{p_t} y_{i,t} - w_t l_{i,t} - \frac{\phi}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 y_t$$

Substituting the demand function and the production function into firm i's profits, the period profit of intermediate good firm i can be rewritten as:

$$\begin{aligned}
d_{i,t} &= \frac{p_{i,t}}{p_t} y_{i,t} - w_t l_{i,t} - \frac{\phi}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 y_t \\
&= \frac{p_{i,t}}{p_t} y_{i,t} - w_t \left(\frac{y_{it}}{x_t}\right)^{\frac{1}{\alpha}} - \frac{\phi}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 y_t \\
&= \left(\frac{p_{i,t}}{p_t}\right)^{1-\varepsilon} y_t - w_t \left(\frac{p_{i,t}}{p_t}\right)^{-\frac{\varepsilon}{\alpha}} \left(\frac{y_t}{x_t}\right)^{\frac{1}{\alpha}} - \frac{\phi}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 y_t
\end{aligned}$$

Hence, the maximisation problem is given by:

$$\max_{p_{i,t+j}} \quad \bar{E}_t \sum_{T=t}^{\infty} \omega^T Q_{t,T} \left\{ \left(\frac{p_{i,T}}{p_T}\right)^{1-\varepsilon} y_T - w_T \left(\frac{p_{i,T}}{p_T}\right)^{-\frac{\varepsilon}{\alpha}} \left(\frac{y_T}{x_T}\right)^{\frac{1}{\alpha}} - \frac{\phi}{2}\left(\frac{p_{i,T}}{p_{i,T-1}} - 1\right)^2 y_T \right\} .$$

Recall that firms are owned by households to equal shares. Firms, therefore, adopt an average of household expectations $\bar{E}_t$. The FOC with respect to $p_{i,T}$ reads:

$$\begin{aligned} 0 =& \omega^T \bar{E}_t \left[Q_{t,T} \Big\{ (1-\varepsilon)\left(\frac{p_{i,T}}{p_T}\right)^{-\varepsilon} \frac{y_T}{p_T} + \varepsilon \left(\frac{p_{i,T}}{p_T}\right)^{-\frac{\varepsilon}{\alpha}-1} \frac{w_T}{\alpha}\left(\frac{y_T}{x_T}\right)^{\frac{1}{\alpha}} \frac{1}{p_T} \right. \\ & \left. -\phi\left(\frac{p_{i,T}}{p_{i,T-1}} - 1\right)\frac{y_{t+j}}{p_{i,T-1}} \right] + \omega^{T+1}\bar{E}_t\left[Q_{t,T+1}\phi\left(\frac{p_{i,T+1}}{p_{i,T}} - 1\right)\frac{y_{T+1}}{p_{i,T}}\frac{p_{i,T+1}}{p_{i,T}} \right] \end{aligned}$$

Setting $T = t$ and using the fact that $Q_{t,t} = 1$, we obtain:

$$\begin{aligned} &(1-\varepsilon)\left(\frac{p_{i,t}}{p_t}\right)^{-\varepsilon}\frac{y_t}{p_t} + \varepsilon\left(\frac{p_{i,t}}{p_t}\right)^{-\varepsilon-1}\frac{w_t}{\alpha}\left(\frac{y_t}{x_t}\right)^{\frac{1}{\alpha}}\frac{1}{p_t} - \phi\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)\frac{y_t}{p_{i,t-1}} \\ &+ \omega\bar{E}_t\left(Q_{t,t+1}\phi\left(\frac{p_{i,t+1}}{p_{i,t}} - 1\right)\frac{y_{t+1}}{p_{i,t}}\frac{p_{i,t+1}}{p_{i,t}}\right) = 0 \end{aligned}$$

Rearranging yields:

$$\begin{aligned} \left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)\frac{p_t}{p_{i,t-1}} =& \frac{\varepsilon}{\phi}\frac{w_t}{\alpha} y_t^{\frac{1-\alpha}{\alpha}} x_t^{-\frac{1}{\alpha}} \left(\frac{p_{i,t}}{p_t}\right)^{-\varepsilon-1} - \frac{(\varepsilon-1)}{\phi}\left(\frac{p_{i,t}}{p_t}\right)^{-\varepsilon} \\ & + \bar{E}_t\left(Q_{t,t+1}\left(\frac{p_{i,t+1}}{p_{i,t}} - 1\right)\frac{y_{t+1}}{y_t}\frac{p_t}{p_{i,t}}\frac{p_{i,t+1}}{p_{i,t}}\right) \end{aligned}$$

This equation states the optimal price setting condition of firm i. Since all firms face the identical problem and form identical expectations, $\bar{E}_t$, they will, in equilibrium, set the same price for their intermediate good, i.e. $p_{i,t} = p_t$. Using the definition of inflation, $\pi_t = \frac{p_t}{p_{t-1}}$, we obtain

$$(\pi_t - 1)\pi_t = \frac{\varepsilon}{\phi}\frac{w_t l_{i,t}^{1-\alpha}}{\alpha x_t} - \frac{(\varepsilon-1)}{\phi} + \omega\bar{E}_t\left[Q_{t,t+1}\frac{y_{t+1}}{y_t}(\pi_{t+1} - 1)\pi_{t+1}\right] .$$

Finally, using (A.3) and the fact that $l_{i,t} = l_t$, we get the New Keynesian Phillips-Curve:

$$(\pi_t - 1)\pi_t = \omega\bar{E}_t\left[Q_{t,t+1}\frac{y_{t+1}}{y_t}(\pi_{t+1} - 1)\pi_{t+1}\right] + \frac{\varepsilon}{\phi}\left(\mathrm{mc}_t^r - \mu\right) \tag{16}$$

where $\mu \equiv \frac{\varepsilon-1}{\varepsilon}$ is the steady state mark-up.

Optimisation under flexible prices. Next, we write down firm's optimisation problem under flexible prices, i.e. given that there are no Rotemberg-costs:

$$\max_{p_{i,t+j},y_{i,t+j}} \quad \bar{E}_t \sum_{j=0}^{\infty} \omega^T Q_{t,T} \left\{ \left(\frac{p_{i,T}}{p_T}\right)^{1-\varepsilon} y_T - w_T \left(\frac{p_{i,T}}{p_T}\right)^{-\frac{\varepsilon}{\alpha}} \left(\frac{y_T}{x_T}\right)^{\frac{1}{\alpha}} \right\}$$

The first order condition for $p_{i,T}$ is given by:

$$\omega^T \mathbb{E}_t \left[Q_{t,T} \left(\left(\frac{p_{i,T}}{p_T}\right)^{-\varepsilon} \frac{y_T}{p_T} + \varepsilon \left(\frac{p_{i,T}}{p_T}\right)^{-\frac{\varepsilon}{\alpha}-1} \frac{w_T}{\alpha} \left(\frac{y_T}{x_T}\right)^{\frac{1}{\alpha}} \frac{1}{p_T} \right) \right] .$$

In a symmetric price equilibrium ($p_{i,T} = p_T$ for all i) and for $T = t$, we get, keeping in mind that $Q_{t,t} = 1$:

$$\frac{1}{p_t} - \varepsilon \mathrm{mc}_t^r \frac{1}{p_t} = 0 \ , \tag{A.4}$$

which gives us

$$\mathrm{mc}_t^r = \frac{\varepsilon - 1}{\varepsilon} \ . \tag{A.5}$$

Hence, under flexible prices, mc_t^r is constant.

A.3. Equilibrium Conditions

The paper develops two models: The model with RE, which serves as baseline case, and the model in which households form expectations with EBL. First, we define the rational expectations equilibrium (REE, henceforth) and second we define equilibrium conditions for the EBL economy. Importantly, we perform aggregation of the cohorts' Euler equation into an aggregate IS-curve following Hagenhoff (2018).

A.3.1. Equilibrium equations: REE

In this appendix, we derive equilibrium conditions. First, we derive $d_t \equiv \frac{\mathcal{D}_t}{p_t}$ from intermediate firm profits:

$$\begin{aligned}
d_t &= \int_0^1 \left\{ \frac{p_{i,t}}{p_t} y_{i,t} - w_t \left(\frac{y_{i,t}}{x_t} \right)^{\frac{1}{\alpha}} - \frac{\phi}{2} \left(\frac{p_{i,t}}{p_{i,t-1}} - 1 \right)^2 y_t \right\} di \\
&= \int_0^1 \frac{p_{i,t}}{p_t} y_{i,t} di - \frac{w_t}{x_t^{\frac{1}{\alpha}}} \int_0^1 y_{i,t}^{\frac{1}{\alpha}} di - \frac{\phi}{2} y_t \int_0^1 \left(\frac{p_{i,t}}{p_{i,t-1}} - 1 \right)^2 di \\
&= \frac{y_t}{p_t^{1-\epsilon}} \int_0^1 p_{i,t}^{1-\epsilon} di - w_t \left(\frac{y_t}{x_t} \right)^{\frac{1}{\alpha}} \int_0^1 \left(\frac{p_{i,t}}{p_t} \right)^{-\frac{\epsilon}{\alpha}} di - \frac{\phi}{2} y_t \int_0^1 \left(\frac{p_{i,t}}{p_{i,t-1}} - 1 \right)^2 di
\end{aligned}$$

Using that with Rotemberg cost the equilibrium is symmetric, the price dispersion term $\Delta_t^p = \int_0^1 \left(\frac{p_{i,t}}{p_t} \right)^{-\frac{\varepsilon}{\alpha}} \approx 1$ is zero. Besides, the price aggregator gives $p_t^{1-\varepsilon} = \int_0^1 p_{i,t}^{1-\varepsilon} di$, so that we get:

$$d_t = y_t - w_t l_t - \frac{\phi}{2} \left(\pi_t - 1 \right)^2 y_t \ , \tag{A.1}$$

where we used that $\left(\frac{y_t}{x_t} \right)^{\frac{1}{\alpha}} = l_t$.

The budget constraint of a household born at $t = k$ is given by:

$$p_t c_{t|k} + b_{t|k} = r_{t-1} \left(b_{t-1|k} + \frac{1-\omega}{\omega} b_{t-1|k} \right) + \bar{Y}_{t|k}$$

where $\bar{Y}_{t|k} = w_t l_{t|k} + \mathcal{D}_t$ denotes household's income from labour and profits. Note that aggregate values are the sum across cohorts, weighted by the respective sizes. For some generic cohort variable $X_{t|k}$, the aggregate value is given by

$$X_t = \sum_{k=-\infty}^{t} \left(1 - \omega \right) \omega^{t-k} X_{t|k} \ .$$

Using this definition and the assumption that the initial wealth of the new born cohort is zero ($b_{t-1|t} = 0$), the aggregate resource constraint is given by:

$$\begin{aligned}\sum_{k=-\infty}^{t} (1-\omega)\,\omega^{t-k}\left(p_t c_{t|k} + b_{t|k}\right) &= \frac{r_{t-1}}{\omega}\sum_{k=-\infty}^{t-1}(1-\omega)\,\omega^{t-k} b_{t-1|k} + \sum_{k=-\infty}^{t}(1-\omega)\,\omega^{t-k}\bar{Y}_{t|k} \\ \Leftrightarrow p_t c_t + b_t &= r_{t-1}\sum_{k=-\infty}^{t-1}(1-\omega)\,\omega^{t-1-k} b_{t-1|k} + p_t w_t l_t + \mathcal{D}_t \\ \Leftrightarrow p_t c_t + b_t &= r_{t-1} b_{t-1} + p_t w_t l_t + \mathcal{D}_t \ , \end{aligned} \tag{A.2}$$

Let $a_t = \frac{b_t}{p_t}$ denote aggregate real financial wealth. Then, the *real* aggregate resource constraint is given by:

$$c_t + a_t = \frac{r_{t-1}}{\pi_t} b_{t-1} + w_t l_t + d_t \ . \tag{A.3}$$

Using the definition of real profits in (A.1), we obtain the aggregate resource constraint

$$y_t = c_t + \frac{\phi}{2}\left(\pi_t - 1\right)^2 y_t \tag{14}$$

Aggregate Consumption Function. In the next two paragraphs we derive the aggregate Euler equation through the aggregate consumption function (Piergallini, 2006). To derive the aggregate consumption function, we first need to obtain cohort-specific consumption. In accordance with the literature, the latter is linear in cohort-wealth. First, derive the aggregate budget constraint of cohort k. The budgets in t and $t+1$ yield:

$$\begin{aligned} &\text{at t:} && p_t c_{t|k} + b_{t|k} = \frac{r_{t-1}}{\omega} b_{t-1|k} + \bar{Y}_{t|k} \ , \\ &\text{at t+1:} && p_{t+1} c_{t+1|k} + b_{t+1|k} = \frac{r_t}{\omega} b_{t|k} + \bar{Y}_{t+1|k} \ , \\ &\text{Combine:} && \frac{b_{t+1|k}}{r_t/\omega} + p_t c_{t|k} + \frac{p_{t+1} c_{t+1|k}}{r_t/\omega} = \frac{r_{t-1}}{\omega} b_{t-1|k} + \bar{Y}_{t|k} + \frac{\bar{Y}_{t+1|k}}{r_t/\omega} \ , \end{aligned}$$

where $\bar{Y}_{t|k}$ is defined as above. Iterating towards infinity and taking expectations gives:

$$\tilde{E}_t^k \sum_{T=t}^{\infty} \omega^{T-t} \mathcal{R}_{t,T} p_T c_{T|k} + \tilde{E}_t^k \left(\lim_{T\to\infty} \omega^T \mathcal{R}_{t,T} b_{T|k}\right) = \frac{r_{t-1}}{\omega} b_{t-1|k} + \tilde{E}_t^k \sum_{T=t}^{\infty} \omega^{T-t} \mathcal{R}_{t,T} \bar{Y}_{T|k} \ ,$$

where $\mathcal{R}_{t,T} = (\prod_{s=t+1}^{T} r_s)^{-1}$. Applying the no-Ponzi condition, $\tilde{E}_t^k \left(\lim_{T\to\infty} \omega^T \mathcal{R}_{t,T} b_{T|k} \right) = 0$, we get:[32]

$$\tilde{E}_t^k \sum_{T=t}^{\infty} \omega^{T-t} \mathcal{R}_{t,T} p_T c_{T|k} = \frac{r_{t-1}}{\omega} b_{t-1|k} + \tilde{E}_t^k \sum_{T=t}^{\infty} \omega^{T-t} \mathcal{R}_{t,T} \bar{Y}_{T|k} \ ,$$

Further, note that

$$\tilde{E}_t^k \sum_{T=t}^{\infty} \omega^{T-t} \mathcal{R}_{t,T} p_T c_{T|k} \cdot \frac{p_{T-1} c_{T-1|k}}{p_{T-1} c_{T-1|k}} \cdot \frac{p_{T-2} c_{T-2|k}}{p_{T-2} c_{T-2|k}} \cdots \frac{p_t c_{t|k}}{p_t c_{t|k}} =$$
$$\tilde{E}_t^k \sum_{T=t}^{\infty} \omega^{T-t} \mathcal{R}_{t,T} \beta^{T-t} Q_{t,T}^{-1} \cdot p_t c_{t|k} = \frac{p_t c_{t|k}}{1 - \beta\omega} \ ,$$

where we used the fact that $\tilde{E}_t^k \left(Q_{t,T|k} \right) = \mathcal{R}_{t,T}$. Finally, it follows that consumption of cohort k is a linear function of its wealth

$$p_t c_{t|k} = \Psi \left(\frac{1}{\omega} r_{t-1} b_{t-1|k} + h_{t|k} \right) \ , \tag{A.4}$$

where $\Psi = (1 - \beta\omega)$ and $h_{t|k}$ denotes household's human wealth, which is given by:

$$h_{t|k} = \tilde{E}_t^k \sum_{T=t}^{\infty} Q_{t,T|k} \omega^{T-t} \bar{Y}_{t|k} \ ,$$

where $\bar{Y}_{t|k} = w_t l_{t|k} + \mathcal{D}_t$. The aggregate consumption function can be obtained by aggregating over the individual consumption function in (A.4) – keeping in mind that the initial financial wealth of the new born generation is equal to zero ($b_{t-1|t} = 0$):

$$\sum_{k=-\infty}^{t} (1-\omega)\, \omega^{t-k} p_t c_{t|k} = \frac{\Psi r_{t-1}}{\omega} \sum_{k=-\infty}^{t-1} (1-\omega)\, \omega^{t-k} b_{t-1|k} + \Psi \sum_{k=-\infty}^{t} (1-\omega)\, \omega^{t-k} h_{t|k} \ ,$$
$$\Leftrightarrow p_t c_t = \Psi r_{t-1} \sum_{k=-\infty}^{t-1} (1-\omega)\, \omega^{t-1-k} b_{t-1|k} + \Psi h_t \ ,$$
$$\Leftrightarrow p_t c_t = \Psi \left(r_{t-1} b_{t-1} + h_t \right) \ . \tag{A.5}$$

[32]From the Euler-equation of a household aged k:

$$\tilde{E}_t^k Q_{t,t+1|k} \equiv \beta \tilde{E}_t^k \frac{c_{t|k}}{c_{t+1|k}} \frac{1}{\pi_{t+1}} = \frac{1}{r_t} = \mathcal{R}_{t,t+1} \ .$$

Note further that $\mathcal{R}_{t,T} = \prod_{s=t+1}^{T} Q_{s-1,s|k} = Q_{t,t+s|k}$. Hence, the term equals the no-Ponzi game condition from the main text:

$$\lim_{T\to\infty} (\beta\omega)^T \tilde{E}_t^k \frac{c_{t|k}}{c_{T|k}} \frac{p_t}{p_T} b_{T|k} = \lim_{T\to\infty} (\beta\omega)^T \tilde{E}_t^k \frac{c_{t|k}}{c_{T|k}} \frac{b_{T|k}}{\pi_T} = \tilde{E}_t^k \left(\lim_{T\to\infty} \omega^T Q_{t,T|k} b_{T|k} \right) \ .$$

Aggregate Euler Equation. Note that $h_{t|k}$ can be written recursively as

$$h_{t|k} = \bar{Y}_{t|k} + \tilde{E}_t^k \left(\omega Q_{t,t+1|k} h_{t+1|k} \right) \ .$$

Aggregating over all cohorts yields,

$$h_t = \bar{Y}_t + \omega \bar{E}_t \left(Q_{t,t+1} h_{t+1} \right) \ ,$$

where we used the definition of aggregate expectations. From (A.2), we know that the aggregate nominal resource constraint is given by:

$$p_t c_t + b_t = r_{t-1} b_{t-1} + p_t w_t l_t + \mathcal{D}_t \ ,$$

whereas the aggregate consumption function has been shown to be:

$$p_t c_t = \Psi \left(r_{t-1} b_{t-1} + h_t \right) \ .$$

Combining these two equations and using the recursive formulation of aggregate human wealth yields:

$$\begin{aligned} p_t c_t &= \Psi \left(p_t c_t + b_t - (p_t w_t l_t + d_t) + h_t \right) \ , \\ &= \Psi \left(p_t c_t + b_t - \bar{Y}_t + \bar{Y}_t + \omega E_t \left(Q_{t,t+1} h_{t+1} \right) \right) \ , \\ &= \Psi \left(p_t c_t + b_t + \frac{\omega}{\Psi} E_t \left(Q_{t,t+1} p_{t+1} c_{t+1} \right) - \omega E_t \left(Q_{t,t+1} r_t b_t \right) \right) \ , \end{aligned}$$

where we used the aggregate consumption function with one-period lead for the third equality. Rearranging yields,

$$\begin{aligned} p_t c_t &= \frac{\omega}{1-\Psi} E_t \left(Q_{t,t+1} p_{t+1} c_{t+1} \right) + \frac{\Psi}{1-\Psi} \left(b_t \frac{r_t}{r_t} - \omega E_t \left(Q_{t,t+1} r_t b_t \right) \right) \ , \\ &= \frac{\omega}{1-\Psi} E_t \left(Q_{t,t+1} p_{t+1} c_{t+1} \right) + \frac{\Psi}{1-\Psi} \left(E_t \left(Q_{t,t+1} r_t b_t \right) - \omega E_t \left(Q_{t,t+1} r_t b_t \right) \right) \ , \\ &= \frac{\omega}{1-\Psi} E_t \left(Q_{t,t+1} p_{t+1} c_{t+1} \right) + \frac{\Psi(1-\omega)}{1-\Psi} E_t \left(Q_{t,t+1} r_t b_t \right) \ , \end{aligned}$$

where the second equality makes use of the definition for $Q_{t,t+1}$. With $\Psi = 1 - \beta\omega$,

$$\begin{aligned} p_t c_t &= \frac{1}{\beta} E_t \left(Q_{t,t+1} p_{t+1} c_{t+1} \right) + \frac{\Psi(1-\omega)}{\beta\omega} E_t \left(Q_{t,t+1} r_t b_t \right) \\ &= \frac{1}{\beta} E_t \left(Q_{t,t+1} p_{t+1} c_{t+1} \right) + \frac{\Psi(1-\omega)}{\beta\omega} b_t \end{aligned}$$

Finally, since real household bond holdings are in zero net supply:

$$a_t = \sum_{k=-\infty}^{t} (1-\omega)\,\omega^{t-k}\frac{b_{t|k}}{p_t} = 0\ ,$$

such that the aggregate Euler equation is given by

$$1 = \frac{1}{\beta}E_t\left(\frac{\pi_{t+1}}{r_t}\frac{c_{t+1}}{c_t}\right)$$

Summary. The total set of equilibrium conditions is given by:

$$\begin{aligned}
y_t &= c_t + \frac{\phi}{2}\left(\pi_t - 1\right)^2 y_t && (14)\\
1 &= \frac{1}{\beta}\bar{E}_t\left(\frac{\pi_{t+1}}{r_t}\frac{c_{t+1}}{c_t}\right) && (A.6)\\
\left(\pi_t - 1\right)\pi_t &= \omega\bar{E}_t\left[Q_{t,t+1}\frac{y_{t+1}}{y_t}\left(\pi_{t+1}-1\right)\pi_{t+1}\right] + \frac{\varepsilon}{\phi}\left(\mathrm{mc}_t^r - \mu\right) && (16)\\
w_t &= \alpha\mathrm{mc}_t^r\frac{y_t}{l_t}\\
w_t &= \psi_n\frac{c_t}{(1-l_t)} && (A.7)\\
y_t &= x_t l_t^{\alpha} && (A.8)\\
r_t &= \bar{r}\left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi}\left(\frac{y_t}{y_t}\right)^{\varphi_y}\exp\left(\epsilon_t^m\right) && (11)\\
x_t &= x^*\exp\left(\epsilon_t^x\right) && (9)\\
\epsilon_t^x &= \rho_x\epsilon_{t-1}^x + \nu_t^x && (10)\\
\epsilon_t^m &= \rho_m\epsilon_{t-1}^m + \nu_t^m\ . && (A.9)
\end{aligned}$$

Thus, we have 8 equations in 8 unknowns $\{y_t, \pi_t, c_t, R_t, \mathbf{w_t}, x_t, \epsilon_t^x, \epsilon_t^m\}_{t=0}^{\infty}$.

Steady State. The zero-inflation ($\pi = 1$) steady state can be summarised by:

$$\begin{aligned}
\pi &= 1 && (A.10)\\
l_{ss} &= 1/3 && (A.11)\\
(9):\ x &= x^* && (A.12)\\
(4):\ r &= \frac{1}{\beta} \overset{(11)}{=} \bar{r} && (A.13)\\
:\ \mathrm{mc}^r &= \mu && (A.14)\\
:\ y &= x^* l_{ss}^{\alpha} && (A.15)\\
(16):\ w &= \mathrm{mc}^r\alpha\frac{y}{l_{ss}} && (A.16)
\end{aligned}$$

$$(2): \; c = y \tag{A.17}$$
$$(10): \; \epsilon^x = 0 \tag{A.18}$$

Linearised equilibrium under Rational Expectations. Next, we linearise equations and reduce the system further. Under rational expectations, $\tilde{E}_t^k \equiv \mathbb{E}_t$ for all cohorts k such that also $\bar{E}_t \equiv \mathbb{E}_t$.

$$\begin{aligned}
\hat{y}_t &= \hat{c}_t \\
\hat{c}_t &= \mathbb{E}_t \hat{c}_{t+1} - (\hat{r}_t - \mathbb{E}_t \hat{\pi}_{t+1}) \\
\hat{\pi}_t &= \beta\omega \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\varepsilon}{\phi} \hat{\mathrm{mc}}_t^r \\
\hat{\mathrm{mc}}_t^r &= \hat{w}_t + l_t - \hat{y}_t \\
\hat{w}_t &= \hat{c}_t + \eta_l \hat{l}_t \\
\hat{l}_t &= \frac{1}{\alpha} (\hat{y}_t - \hat{x}_t) \\
\hat{r}_t &= \varphi_\pi \hat{\pi}_t + \varphi_y \hat{y}_t + \hat{\epsilon}_t^m \\
\hat{x}_t &= x^* \hat{\epsilon}^x \\
\hat{\epsilon}_t^x &= \rho_x \hat{\epsilon}_{t-1}^x + \nu_t^x \\
\hat{\epsilon}_t^m &= \rho_m \hat{\epsilon}_{t-1}^m + \nu_t^m \quad .
\end{aligned}$$

where $\eta_l \equiv \frac{l}{1-l}$ denotes the stationary labour-leisure share. Use the aggregate resource constraint and the rewritten MRS between labour and consumption to eliminate $\hat{c}_t$ and $\hat{w}_t$. Then, we get a standard New Keynesian model:

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - (\hat{r}_t - \mathbb{E}_t \hat{\pi}_{t+1}) \tag{A.19}$$
$$\hat{\pi}_t = \beta\omega \mathbb{E}_t \hat{\pi}_{t+1} + \kappa (\hat{y}_t - \hat{x}_t) \tag{A.20}$$
$$\hat{r}_t = \varphi_\pi \hat{\pi}_t + \varphi_y \hat{y}_t + \hat{\epsilon}_t^m \tag{A.21}$$
$$\hat{x}_t = x^* \hat{\epsilon}_t^x \tag{A.22}$$
$$\hat{\epsilon}_t^x = \rho_x \hat{\epsilon}_{t-1}^x + \nu_t^x \tag{A.23}$$
$$\hat{\epsilon}_t^m = \rho \hat{\epsilon}_{t-1}^m + \nu_t^m \; , \tag{A.24}$$

where $\kappa = \frac{\varepsilon(1+\eta_l)}{\alpha\phi}$.

A.3.2. Euler Equation Learning and Equilibrium equations under EBL

Euler Equation Learning. To derive the equilibrium conditions under EBL, we rely on the widely used assumption in the SL-literature that agents choose consumption plans that satisfy the associated Euler equations. Under this so called *Euler equation learning*, agents base their decisions on the time t trade-off between current consumption and next period's consumption given the sequential budget constraint and on the one period ahead forecast of consumption. Intuitively, each period agents equate the marginal costs of postponing consumption with the benefits, taking into account their current budget constraint and their one period ahead forecast of consumption and inflation.

Evans et al. (2013) argue that this formulation of optimisation behaviour is plausible. First, a forecast of own consumption in $t+1$ is required to make the period t consumption choice. It is helpful to consider what happens under REs. In a model with REs, the endogenous variables are a function of the relevant state variables in the economy. In our model, however, agents do not know the RE equilibrium mapping and are unaware of the consumption function specified under REs. Instead, agents try to learn the RE mapping. Since the time t trade-off requires agents to form an expectations about tomorrow's consumption plan without the knowledge of the RE mapping, the agent internalises how tomorrows consumption evolves given her PLM in mind. Given this PLM, the best "decision" of consumption tomorrow is plausibly a linear function of current consumption. Further, agents are assumed to think just one period ahead without explicitly taking into account the inter-temporal budget constraint. Yet, Evans et al. (2013) show that for a convergent learning process, the inter-temporal budget is satisfied. Since along the sequence of *temporary* equilibria, agents consumption is equal to its income and by imposing a solvency condition like the one in equation (3), ex post consistency in the accounting over the planing horizon of the household that has some strictly positive probability to live forever is fulfilled. For further details see Evans and Honkapohja (2012).

Convergence of the learning process is an important property of the SL-literature. Usually, agents learn based on a correctly-specified PLM so that one can derive the conditions for convergence to the RE equilibrium (called *E-stability*). Yet, agents in our model learn based on a misspecified model of the economy, so that convergence to the REE is not possible. Instead, Evans and Honkapohja (2012) and Berardi (2009) discuss the concept of a restricted perceptions equilibrium and of a heterogeneous expectations equilibrium, respectively. Both concepts describe the situation in which agents beliefs converged to a fixed point that, while not being the REE, allows them to make the best possible forecast given the (misspecified) model they rely on. In our model, expectations are not only heterogeneous but also based on a wrong PLM. Yet, through simulation one can show that PLM parameters converge to a non-explosive value for old agents (see figure 6) so that expectations are stable.

Aggregation. We follow the literature on heterogeneous expectations that relies on the axiomatic approach of Branch and McGough (2009) in order to be able to properly aggregate the decisions of agents with heterogeneous expectations without including the wealth distribution as an additional state variable.[33] In order to do so, we need to rely on two key assumptions from Branch and McGough (2009):

1. The structure of higher order beliefs: $\tilde{E}_t^i \tilde{E}_t^k x_{t+1} = \tilde{E}_t^i x_{t+1},\ i \neq k$.
2. Agents expect to return to the same wealth in the long-run: $\tilde{E}_t^i \left(\hat{c}_\infty - \hat{c}_\infty^i\right) = 0$.

Consider again the Euler equation given in (4):

$$c_{t|k} = \beta \tilde{E}_t^k \left(c_{t+1|k} \frac{r_t}{\pi_{t+1}} \right)$$

The linearised Euler equation from a household in cohort i is given by:

$$\hat{c}_{t|i} = \tilde{E}_t^i \hat{c}_{t+1|i} - \left(\hat{r}_t - \tilde{E}_t^i \hat{\pi}_{t+1} \right)\ \forall i \ .$$

Forward iteration of the Euler equation yields:

$$\hat{c}_{t|i} = \underbrace{\lim_{j\to\infty} \tilde{E}_t^i \hat{c}_{\infty|i}}_{\equiv \tilde{E}_t^i \hat{c}_\infty^i} - \tilde{E}_t^i \sum_{j=0}^{\infty} \left(\hat{r}_{t+j} - \hat{\pi}_{t+1+j} \right)\ \forall i \ . \tag{A.25}$$

where we used Assumption A5. of Branch and McGough (2009) which states that the Law of Iterated Expectations is satisfied, i.e. $\tilde{E}_t^k \left(\tilde{E}_{t+1}^k \left(c_{t+2} \right) \right) = \tilde{E}_t^k (c_{t+2})$.
The aggregated linearised resource constraint in t and in $t+1$:

$$\begin{aligned} \hat{c}_t &= (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \hat{c}_{t|k} = \hat{y}_t \\ \hat{c}_{t+1} &= (1-\omega) \sum_{k=-\infty}^{t+1} \omega^{t+1-k} \hat{c}_{t+1|k} = \hat{y}_{t+1} \end{aligned} \tag{A.26}$$

Next, insert the forward iterated Euler equation (A.25) into the $t+1$-resource (A.26) for $\hat{c}_{t+1|k}$ (for each cohort in $t+1$, respectively) and take expectations of cohort i:

$$\tilde{E}_t^i \left[(1-\omega) \sum_{k=-\infty}^{t+1} \omega^{t+1-k} \left(\tilde{E}_t^k \hat{c}_\infty^k - \tilde{E}_t^k \sum_{j=1}^{\infty} \left(\hat{r}_{t+j} - \hat{\pi}_{t+1+j} \right) \right) \right] = \tilde{E}_t^i \left(\hat{y}_{t+1} \right) \tag{A.27}$$

[33] Examples in the literature that rely in the axiomatic approach of Branch and McGough (2009) are for example Gasteiger (2014), Di Bartolomeo et al. (2016), Hagenhoff (2018).

Following Branch and McGough (2009) and the exposition in Hagenhoff (2018) we note that how one thinks of higher-order beliefs matters for the further steps to aggregation. The former impose that agents' expectations about what other agents expect, are equal to their own expectation, which corresponds to assumption 1. Having departed from RE and, therefore, having assumed that agents do not know the underlying structure of the economy, imposing that they can not foresee how others form expectations is a natural step. Under assumption 1, we can rewrite (A.27) as:

$$\begin{aligned}
\tilde{E}_t^i\left(\hat{y}_{t+1}\right) &= (1-\omega)\sum_{k=-\infty}^{t+1}\omega^{t+1-k}\left(\tilde{E}_t^i c_\infty^k - \tilde{E}_t^i\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right)\right)\\
&= \tilde{E}_t^i\left((1-\omega)\sum_{k=-\infty}^{t+1}\omega^{t+1-k}c_\infty^k\right)\\
&\quad -\tilde{E}_t^i\left((1-\omega)\sum_{k=-\infty}^{t+1}\omega^{t+1-k}\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right)\right)\\
&= \tilde{E}_t^i c_\infty - \tilde{E}_t^i\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right)
\end{aligned}$$

where the last equality uses assumption 2, which we discuss below, and that weights sum to one. We use this to substitute the infinite sum of real interest rates in (A.25).

$$\begin{aligned}
\hat{c}_{t|i} &= \tilde{E}_t^i c_\infty^i - \tilde{E}_t^i\sum_{j=1}^{\infty}\left(\hat{r}_{t+j}-\hat{\pi}_{t+1+j}\right) - \left(\hat{r}_t - \tilde{E}_t^i\hat{\pi}_{t+1}\right)\\
&= \tilde{E}_t^i\left(\hat{y}_{t+1}\right) - \tilde{E}_t^i\left(c_\infty - c_\infty^i\right) - \left(\hat{r}_t - \tilde{E}_t^i\hat{\pi}_{t+1}\right)\ ,
\end{aligned}$$

Of particular interest is the term $\tilde{E}_t^i\left(c_\infty - c_\infty^i\right)$, which denotes expected differences of own consumption and aggregate household consumption in the limit. Branch and McGough (2009) deal with such a term by assuming that agents agree on expected differences in limiting consumption so that in aggregation it vanishes. Equivalently, Hagenhoff (2018) assumes that agents expect to be back at the steady state in the long-run which also eliminates the term. We use the same assumption but adopt it for our usage in the following sense: take cohorts, i and k that both expect to have steady state consumption in the long-run:

$$\tilde{E}_t^j\hat{c}_\infty^j = \tilde{E}_t^j\hat{c}_\infty \qquad \forall\ j = i,k$$

Now, let cohort i take expectations of the limiting expectations for cohort k and invoke assumption 1: $\tilde{E}_t^i\left(\tilde{E}_t^k\hat{c}_\infty^k\right) \overset{2}{=} \tilde{E}_t^i\left(\tilde{E}_t^k\hat{c}_\infty\right) \overset{1}{=} \tilde{E}_t^i\hat{c}_\infty$. Thus, cohorts not only expect to be back at the steady state but expect this for others as well. Assuming agents expect to be back at the steady state in the long run, in which all consume equally, and having a unit mass of agents implies $c = c^k \; \forall \; k$, holds for non-explosive PLM-parameters. Under assumption 2:

$$\hat{c}_{t|i} = \tilde{E}_t^i \left(\hat{y}_{t+1}\right) - \left(\hat{r}_t - \tilde{E}_t^i \hat{\pi}_{t+1}\right) \; . \tag{A.28}$$

Note that (A.28) holds for all cohorts. Insert into the aggregate resource constraint in t:

$$\begin{aligned}
\hat{y}_t &= (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \hat{c}_{t|k} \\
&= (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \left(\tilde{E}_t^k \left(\hat{y}_{t+1}\right) - \left(\hat{r}_t - \tilde{E}_t^k \hat{\pi}_{t+1}\right)\right) \\
&= (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \tilde{E}_t^k \left(\hat{y}_{t+1}\right) - (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \left(\hat{r}_t - \tilde{E}_t^k \hat{\pi}_{t+1}\right) \; .
\end{aligned}$$

Finally, we use the definition of aggregate expectations, $\bar{E}_t x_{t+1} = (1-\omega) \sum_{k=-\infty}^{t} \omega^{t-k} \tilde{E}_t^k x_{t+1}$ to receive the aggregate dynamic IS-curve:

$$\hat{y}_t = \bar{E}_t \left(\hat{y}_{t+1}\right) - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1}\right) \; . \tag{A.29}$$

Note that the derivation of the New Keynesian Phillips Curves stays unchanged.

Summary. To sum up, we receive the following system of equations:

$$\hat{y}_t = \bar{E}_t \hat{y}_{t+1} - \left(\hat{r}_t - \bar{E}_t \hat{\pi}_{t+1}\right) \tag{A.30}$$

$$\hat{\pi}_t = \beta\omega \bar{E}_t \hat{\pi}_{t+1} + \kappa \left(\hat{y}_t - x^* \hat{\epsilon}_t^x\right) \tag{A.31}$$

$$\hat{r}_t = \varphi_\pi \hat{\pi}_t + \varphi_y \hat{y}_t + \epsilon_t^m \tag{A.32}$$

$$\hat{\epsilon}_t^x = \rho_x \hat{\epsilon}_{t-1}^x + \nu_t^x \tag{A.33}$$

$$\hat{\epsilon}_t^m = \rho \hat{\epsilon}_{t-1}^m + \nu_t^m \; , \tag{A.34}$$

where the expectations in the NKPC and in the dynamic IS-curve follow EBL.

B. Implied weights on past data

To see how figure 3 is calculated, we derive the implied weights an 50 year old individual puts on past data observed in his life-time to form expectations on a generic variable x_t. Assume that the individual is born at time k such that $age \equiv t - k = 200$, i.e. at time t, the individual is 50 years old. Multiplying equation (20a) by $R^x_{t|k}$ on both sides:

$$\begin{aligned} R^x_{t|k}\delta^x_{t|k} &= R^x_{t|k}\delta^x_{t-1|k} + \gamma_{t|k}v_{t-1}(x_{t-1} - \delta^{x\prime}_{t-1|k}v_{t-1}) \\ &= (R^x_{t|k} - \gamma_{t|k}v_{t-1}v'_{t-1})\delta^x_{t-1|k} + \gamma_{t|k}v_{t-1}x_{t-1} \\ &\overset{(20b)}{=} (1-\gamma_{t|k})R^x_{t-1|k}\delta^x_{t-1|k} + \gamma_{t|k}v_{t-1}x_{t-1} \ . \end{aligned} \tag{B.1}$$

Iterating forward on equation (B.1), yields

$$\begin{aligned} R_{t|k}\delta^x_{t|k} &= (1-\gamma_{t-1|k})(1-\gamma_{t|k})R^x_{t-2|k}\delta^x_{t-2|k} + \gamma_{t-1|k}(1-\gamma_{t|k})v_{t-2}x_{t-2} + \gamma_{t|k}v_{t-1}x_{t-1} \\ &= \dots \\ &= V'\Omega X \ , \end{aligned} \tag{B.2}$$

where Ω is a diagonal matrix, V is a matrix that entails the stacked regressor vectors, v_{t-1-s}, and X collects the stacked dependent variable x_{t-s} for $s \in \{0, \dots, age\}$. In a similar way, rewriting equation (20b) and iterating forward yields:

$$\begin{aligned} R^x_{t|k} &= R^x_{t-1|k} + \gamma_{t|k}(v_{t-1}v'_{t-1} - R^x_{t-1|k}) \\ &= (1-\gamma_{t|k})R^x_{t-1|k} + \gamma_{t|k}v_{t-1}v'_{t-1} \\ &= \dots \\ &= V'\Omega V \ . \end{aligned} \tag{B.3}$$

Combining equation (B.2) and (B.3) yields:

$$\delta^x_{t|k} = (R^x_{t|k})^{-1}R^x_{t|k}\delta^x_{t|k} = (V'\Omega V)^{-1}V'\Omega X \ . \tag{B.4}$$

As can be seen in equation (B.4), the EBL scheme is a recursive *weighted*-least squares estimator where the weighting matrix Ω is a diagonal matrix whose diagonal elements can be formulated recursively as:

$$\omega_{t|k}(s) = \omega_{t|k}(s-1)\frac{1-\gamma_{t-s+1|k}}{\gamma_{t-s+1|k}}\gamma_{t-s|k} \ , \tag{B.5}$$

with initial condition $\omega_{t|k}(-1) = \frac{\gamma_{t+1|k}}{1-\gamma_{t+1|k}}$. Note, that the left panel of figure 3 is created using equation (B.5).

C. Algorithm

As described in section 3, the vector of all endogenous variables, Y, contains a subset variables that appear with lead, Y^f. Households in cohort k forecast future values of variables in Y^f using the following linear regression model:

$$\mathbf{z}_t = \delta^{\mathbf{z}}_{t-1|k}\mathbf{z}_{t-1} + \varepsilon_{t|k} \quad \text{for each } \mathbf{z} \in Y^f \tag{C.1}$$

where $\delta^{\mathbf{z}}_{t-1|k}$ is estimated via recursive least squares according to:

$$\delta^{\mathbf{z}}_{t|k} = \delta^{\mathbf{z}}_{t-1|k} + \gamma_{t|k}\left(R^{\mathbf{z}}_{t|k}\right)^{-1}\mathbf{z}_{t-1}\hat{\varepsilon}^{\mathbf{z}}_{t|k} \tag{20a}$$

$$R^{\mathbf{z}}_{t|k} = R^{\mathbf{z}}_{t-1|k} + \gamma_{t|k}(\mathbf{z}_{t-2}\mathbf{z}'_{t-2} - R^{\mathbf{z}}_{t-1|k})\ . \tag{20b}$$

where $\hat{\varepsilon}^{\mathbf{z}}_{t|k} = \mathbf{z}_t - \delta^{\mathbf{z}}_{t-1|k}\mathbf{z}_{t-1}$ for each $x \in Y^f$.

Algorithm

1. Simulate the exogenous process w_t.[34]
2. Simulate the economy under RE for an initial phase of 120 periods to get initial values $\delta^{\mathbf{z}}_{-1|k}$ and $R^{\mathbf{z}}_{-1|k}$.[35]
3. In period $T^b + 1$, we endow all cohorts with the same δ and R and switch to EBL.
4. We insert the PLM parameters into the mod-file of the EBL economy in which expectations are replaced by the PLMs. `Dynare` gives us policy functions.
5. After solving the model, the economy is simulated one further period to obtain new observations for each $\mathbf{z} \in Y^f$ and v by using the policy functions obtained in 4.
6. Based on the new observation we update $\delta^{\mathbf{z}}$ and $R^{\mathbf{z}}$ using RLS for very cohort and PLM. The algorithm then starts again at 4. for T periods.

The actual simulation displayed in the graphs discards the first iterations under EBL to allow the impact of initial values from the RE economy to die out. When we use `dynare`, we need to specify the number of cohorts in the mod-file. While, theoretically, agents can live for infinity (and number of cohorts is infinite), we need to choose a finite number when specifying the model. A high number approximates the "true" economy closer but comes at the cost of computational complexity. We, therefore, choose the number of cohorts as

[34] For the simulation we rely on Matlab and `dynare 4.5.2`.

[35] They follow from a simple least squares regression of inflation (output) on its lagged value. In that sense initial beliefs are close to those implied by the RE model.

200 (equivalent to a 50-year working life) and normalise cohorts weights to sum to one. We endow the new born cohort with PLM parameters drawn from a truncated normal distribution around the RE-estimate which serves to start the updating recursion via RLS.

In step 6., we use a so-called projection facility (PF, henceforth) to ensure the model with EBL can be solved. Conceptually, it reinitializes the updating step as soon as new simulated data implies unstable PLM parameters. We proceed in 2 steps:

Projection Facility

1. We take the updated PLM coefficients and check whether they make they forecasting model explosive, $|\delta^{\mathbf{z}}_{t|k}| > 1$. If the forecasting model generates non-explosive behaviour, we allow the updating step.

2. If behaviour is explosive we proceed as follows:

   (a) The problem may lie with the "effective gain" $\gamma_{t|k}(R^{\mathbf{z}}_{t|k})^{-1}$: updating in (20a) may also for a small gain $\gamma_{t|k}$ lead to explosive values if the matrix $R^{\mathbf{z}}$ has its smallest eigenvalue close to zero. As outlined by Slobodyan and Wouters (2012a) this might occur when initial beliefs are derived from the REE. We follow the authors and invoke a Ridge correction for those cases: if the smallest eigenvalue of $R^{\mathbf{z}}$ is smaller than a constant ι, the inverse of the moment matrix $(R^{\mathbf{z}})^{-1}$ is replaced by $(R^{\mathbf{z}} + \iota\mathbf{I})^{-1}$.[36]

   (b) If either the smallest eigenvalue of $R^{\mathbf{z}}$ did not invoke above correction or if despite correction a household's forecasting model is explosive, we invoke the "standard" form of the PF and ignore the updating step.

Two comments are at order. First, our usage of the PF deviates from the "standard" procedure that ignores the updating step if behaviour is explosive. Any time the PF is used, information and agent behaviour (generated by our model) are discarded. Using the correction for the effective gain before the "standard" PF, constitutes less of an intervention into the model. Furthermore, although our gain parameter is decreasing, young agents, by construction, have high gain parameters. Thus, in case the forecast error turns out to be extremely high (e.g. due to a high shock), especially estimates of young agents are easily drifting to extreme PLM parameters. While we desire strong updating behaviour of young agents, we face the trade-off against having extreme PLM parameters. Therefore, we still invoke the PF that ignores updating steps that lead to parameters ± 1. Second, using the PF can be rationalized by (realistically) assuming that forecasters do not use explosive models. Otherwise, forecasts would deviate greatly from actual outcomes, so that agents want to switch to a more appropriate PLM.

[36]We follow Slobodyan and Wouters (2012a) and set $\iota = 10^{-5}$.

D. The Effect of the Shape Parameter θ on Dynamic Properties of EBL

The gain to new information is age-dependent under EBL. To parametrize its behaviour across age-groups we use the estimated shape parameter θ of Malmendier and Nagel (2016). While we set $\theta = 3.044$ to the estimate of their preferred specification, the authors also provide values for different regression specifications. In fact our choice of θ is the lowest value provided so that we check the robustness of our results against the highest estimate in Malmendier and Nagel (2016), $\theta = 4.144$.

Recall that the shape parameter governs how quickly gains reduce across age groups (see Figure 3). A higher value, hence, will intensify the difference between young and old agents, which is the root of expectation differences and of our results. Figure 13 compares the distribution of aggregate PLM parameters under both specifications of θ. As expected, we observe dispersion of parameters to increase as the gain is set to the highest value estimated by Malmendier and Nagel (2016). In that sense, our results constitute a lower bound.

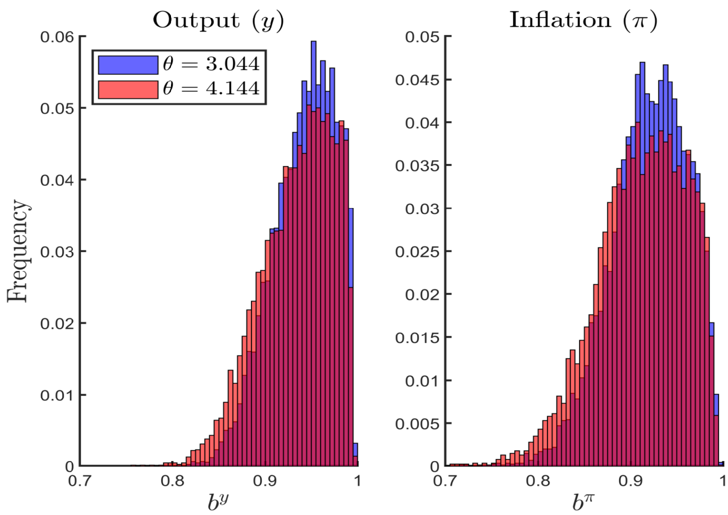

FIGURE 13: Distribution of PLM Parameters in the Model with EBL

Notes: The blue histogram denotes the distribution of PLM parameters in the model under EBL while the red histogram shows the distribution of the PLM parameters under CGL. The left part shows the PLM parameter for output while the right part shows the PLM parameter for inflation. We simulate the economy with EBL and CGL for 10,000 periods.

E. The Effect of EBL on Monetary Policy

E.1. Stabilisation Trade-Off of Monetary Policy

Recall that the shift of the policy frontier under EBL for an increase in the mass of young agents was mainly driven by the experience channel, that is the impact of lower aggregate perceived persistence on output and inflation in a young economy.

The left upper panel of Figure 14 shows the full shift in the policy frontier under EBL and RE for the baseline (old) and young economies. The upper right panel additionally includes a model with CGL. Importantly, under the CGL framework all agents update equally and, hence, no effect from experience arises. Comparing the position of the dotted blue lines in both panels to the red dotted line that denotes the young RE economy, we see that without experience effects (CGL) the policy frontier shift is comparable to the one occurring under RE. The lower two panels provide the same results but for a stronger focus on inflation stabilisation. Our results are robust to a variation of the parameter.

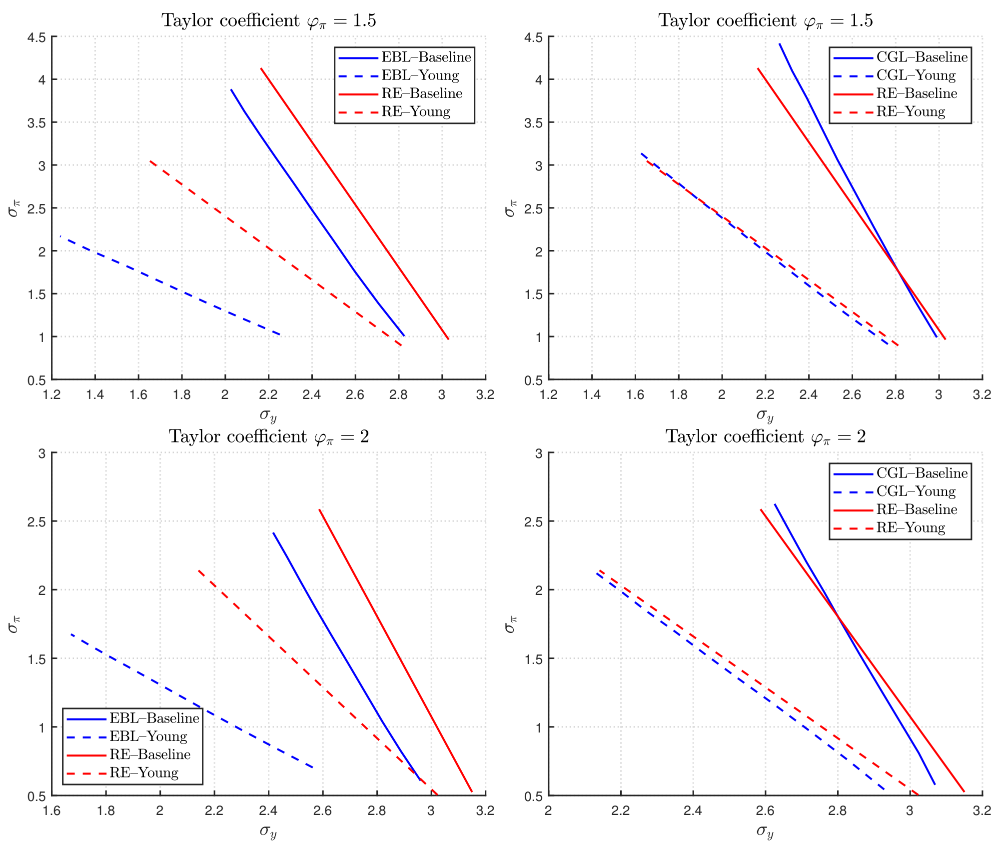

FIGURE 14: Monetary Policy Trade-off under a TFP–Shock under RE and CGL

Notes: The left panel compares the monetary policy trade-off for under EBL (red lines) relativ to the one in the model with RE (blue lines). The right panel compares the monetary policy trade-off in the model with learning when we shut off the experience effects (red lines) relativ to the one in the model with RE (blue lines). In each case, the solid line displays the baseline economy ($\omega = 0.995$) while the dotted lines display the young economy ($\omega = 0.900$).

E.2. Higher Stabilisation Motive for Inflation

Figures 15 and 16 repeat the analysis in the main text but for a higher focus in inflation stabilisation. Our core results are not affected as can be seen by the relative positions of the policy frontiers. For a higher focus on inflation stabilisation the intersection of the policy frontiers for RE and CGL occurs later, since the interest rate is even for comparatively high values on φ_y still strongly affected by inflation. Given RE factor in the impact of current and future nominal rate changes on output, the volatility of output is higher for more values of φ_y.

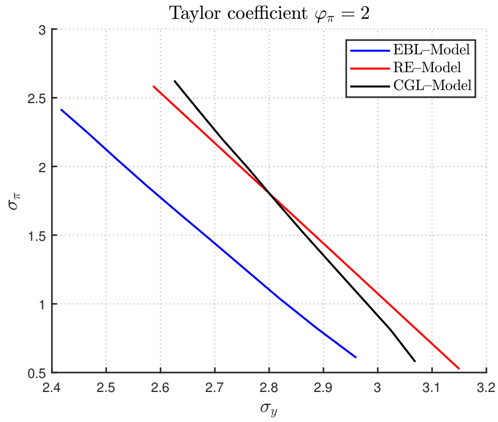

FIGURE 15: Monetary Policy Frontier under a TFP–shock

Notes: We define a grid of points for φ_y. We then simulate the economy as described above for $10,000$ periods for each grid point while holding $\varphi_\pi = 1.5$. We then plot the standard deviation of output against the one for inflation for the models under EBL (blue), CGL (black) and with RE (red).

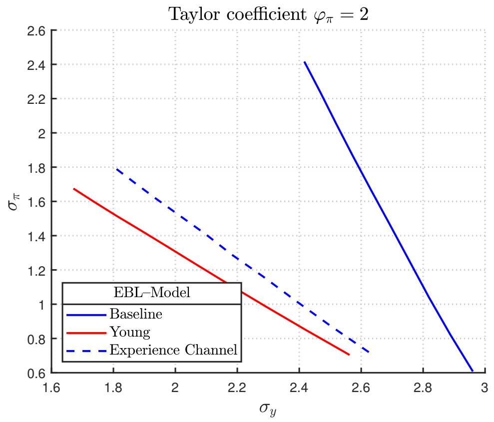

FIGURE 16: Monetary Policy Frontier under a TFP–shock

Notes: In addition to the remarks on Figure 9: we show the frontiers for the models under EBL in an old economy (blue solid) and in a young economy. The latter is divided into the full effect (red) and into the effect from experiences alone (blue dashed).

E.3. Transmission of Monetary Policy

In the main text we discuss how a change in the mass of young agents affects the response to monetary policy shocks. As can be seen from comparing the first and second row in Figure 17, the effect of a change in ω on the IRFs under RE is negligible.

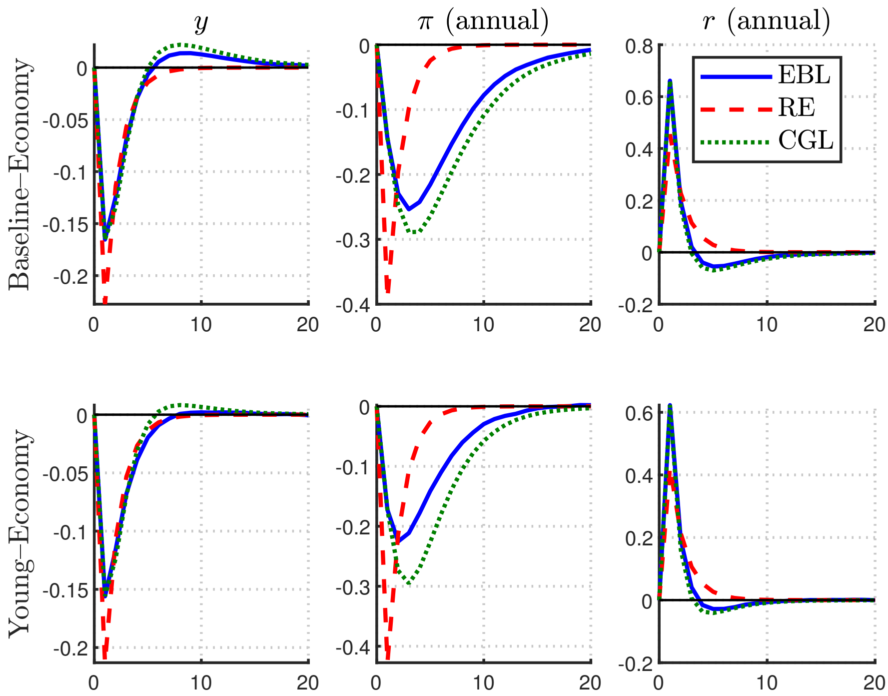

FIGURE 17: IRF to a MP shock in Model with RE

Notes: Impulse response functions to a monetary policy shock in the model with RE. The blue solid lines denote the IRF in the baseline model ($\omega = 0.995$) and the red dashed lines denote the IRF in the young economy ($\omega = 0.900$). Output is measured as percentage deviation from the steady state while the other variables are measured as (annualised) deviation from their respective steady state.

Furthermore, Figure 17 compares the effect of the age-distribution on the IRF in a model without the Experience Channel (all cohorts have the same expectations on output and inflation) to the model with EBL. As discussed in Section 4.2 the aggregate perceived persistence is lower under the latter assumption on expectation formation. When the share of young agents increases under EBL, the aggregate perceived persistence falls again compared to the baseline (old) economy. In contrast, under CGL without experience effects, such changes do not occur. In consequence, the difference in IRFs is more pronounced when considering the young economy in the second line of Figure 17.

E.4. The Effect of Experiences on The Monetary Policy Trade-off

In the main text we compare the policy frontier under EBL to those under RE and CGL. The latter assumption also imposes adaptive expectations like EBL and, methodology, is similar. Yet, in contrast to EBL, it is impossible to have effects through experiences. Figure 18 depicts policy frontiers for models with different types of expectation formation. Importantly, the solid blue line shows our baseline EBL case in which the gain of agents, depending on age, ranges from $\frac{\theta}{K}$ to 1, where θ is the EBL shape parameter and K the maximum number of cohorts entering aggregation. If in contrast, we choose models based on CGL with either parameter choice (red and black lines), we find the EBL policy frontier to lie in between both. This is to be expected, since the *average* gain under EBL is an average across age groups. In fact, with a CGL parameter of $g^* = 0.15$ we can replicate the EBL frontier with a CGL expectation formation (light blue). Yet, this does not invalidate the usage of EBL for two reasons. First, such a value for the gain parameter is empirically implausible (see Milani (2007) and Slobodyan and Wouters (2012b)). Second, such an economy features no Experience Channel, which is the essential ingredient that guides effects through demographic changes.

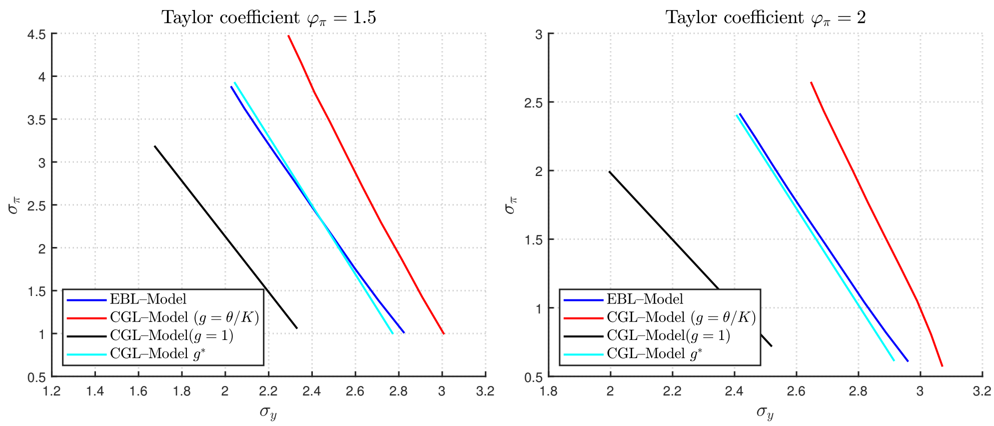

FIGURE 18: Monetary Policy Frontier under a TFP-shock

Notes: The figure shows the Policy Frontier in the model under the baseline calibration under (a) EBL, (b) CGL when the gain parameter is set to $g \simeq 1$ (only young agents); (c) CGL when the gain parameter is set to $g = \theta/K$ (only old agents); and (d) when the gain parameter is set such that it comes close to the Policy Frontier under EBL ($g^* = 0.15$). The left (right) panel shows the Policy frontier when the Monetary Policy parameter is set to $\varphi_\pi = 1.5$ ($\varphi_\pi = 2$).